# A Proactive Dynamic Spectrum Access Method against both Erroneous Spectrum Sensing and Asynchronous Inter-Channel Spectrum Sensing

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## Abstract

Most of the current frequency hopping (FH) based dynamic spectrum access (DSA) methods concern a reactive channel access scheme with synchronous inter-channel spectrum sensing, i.e., FH is reactively triggered by the primary user (PU)'s return reported by spectrum sensing, and the PU channel to be switched to is assumed precisely just sensed or ready to be sensed, as if the inter-channel spectrum sensing moments are synchronous. However, the inter-channel spectrum sensing moments are more likely to be asynchronous, which risks PU suffering more interference. Moreover, the spectrum sensing is usually erroneous, which renders the problem more complex. To address this problem, we propose a proactive FH based DSA method against both erroneous spectrum sensing and asynchronous inter-channel spectrum sensing (moments). We term it as proactive DSA. The optimal FH sequence is obtained by dynamic programming. The complexity is also analyzed. Finally, the simulation results confirm the effectiveness of the proposed method.

***Keywords:*** DSA, FH, cognitive radio, asynchronous inter-channel spectrum sensing, erroneous spectrum sensing

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## 1. Introduction

Cognitive radio (CR) enables the unlicensed use of free licensed bands to use the radio spectrum efficiently, and this has led CR to a prominent position in the field of wireless communication [1][2]. When a secondary user (SU) undertakes its task of data transmission in complex radio environments, it should not interfere with the communication of the primary user (PU). Thus, the design of dynamic spectrum access (DSA) methods become very challenging [3].

Usually, the PU channel is described by the Hidden Markov Model (HMM) in the prevailing literature. Many DSA methods have been developed based on the HMM. Ahmad et al. [4] use a POMDP model [5] for correlated PU channels and efficiently execute the data transmission task with a myopic policy for SU throughput maximization. If PU channels are independent of each other, a multi-armed bandit problem is formulated [6], which is also solved by a myopic policy. Moreover, the Markov Model is also adopted for DSA in designing an efficient MAC protocol [7], system capacity improvement [8], channel allocation of multimedia application [9], and network security [10].

Although the DSA methods in [4][5][6][7][8][9][10] are well developed, they just concern one specific performance metric while the other metrics, such as the transmission rate and power, are not considered. Kim et al. [11] jointly considers these parameters in DSA as a constrained optimization problem, but spectrum sensing and DSA are separately investigated. In fact, spectrum sensing and DSA are interconnected, so they should be investigated together. Hence, several cross layer approaches have been developed to improve the performance of either spectrum sensing [13], or DSA [14], or even both [15]. The aforementioned methods are optimal or nearly optimal regarding their respective problem assumptions and goals. However, most of them concern a reactive channel access manner with synchronous inter-channel spectrum sensing, i.e., FH is reactively triggered by the PU's return reported by spectrum sensing and the PU channel to be switched to is considered as precisely just sensed or ready to be sensed, as if the inter-channel spectrum sensing moments are synchronous, as shown in **Fig.1-(a)**. We term this kind of method as synchronous DSA. However, the corresponding inter-channel spectrum sensing moments are more likely to be asynchronous, which is universal, but rarely considered. According to synchronous DSA, we term the DSA method that operates with asynchronous inter-channel spectrum sensing moments and reactively triggers FH via the PU's return reported from spectrum sensing as passive DSA as shown in **Fig.1-(b)**.

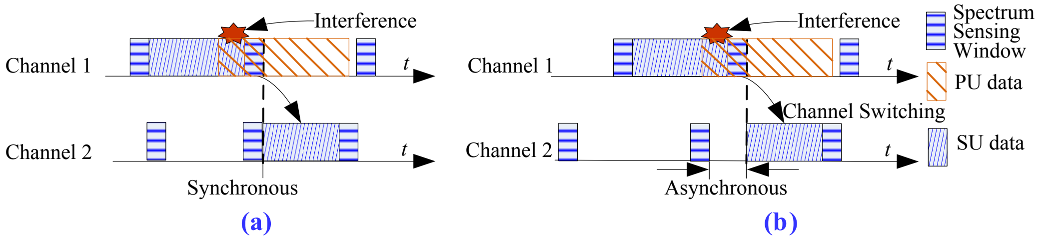

**Fig. 1.** Different cases of channel switching **(a)** channel switching with synchronous inter-channel spectrum sensing; **(b)** the corresponding asynchronous case.

There are several problems with passive DSA. First, when FH is reactively triggered by the PU's return reported from spectrum sensing, the interference has already occurred. Next, since in most cases the probability of the PU's return is monotonically increasing [13], the asynchronicity of inter-channel spectrum sensing moments risks PU suffering more interference, which can make PU unable to work in the interference-sensitive applications. Finally, the spectrum sensing is usually erroneous, which renders the channel access unwise. PU would suffer from additional interference from SU due to this unwise channel access. Even though proactive channel access is studied in [16], interference is not sufficiently reduced and erroneous spectrum sensing is not considered; moreover, it suffers from the same problem when the inter-channel spectrum sensing moments are asynchronous as passive DSA does. Asynchronicity is considered for spectrum sensing, especially the cooperative case [17][18]. In [19], asynchronous spectrum sensing is concerned for dynamic spectrum access; however, a reactive FH policy is employed thus the risk that PU would suffer more interference due to asynchronous inter-channel spectrum sensing has not yet been reduced.

Therefore, we propose a proactive DSA method against both erroneous spectrum sensing and asynchronous inter-channel spectrum sensing (moments). The varying of the PU channel between "busy" and "idle" is modeled as a continuous-time alternating renewal process. The erroneous spectrum sensing is accounted for by a detection probability. In particular, we propose that FH is proactively triggered by the emergence of another channel with a lower probability of PU's existence compared to that of the current channel in use by the SU. The optimal FH sequence is obtained via dynamic programming. The complexity of the proposed method is also provided. The simulation results confirmed the interference reduction ability of the proposed method.

The contributions of this paper are as follows:

- The erroneous spectrum sensing is investigated for DSA. The design of a spectrum sensing method with high detection probability is challenging. Considering erroneous spectrum sensing is more practical than assuming perfect spectrum sensing.
- The inter-channel spectrum sensing moments are considered asynchronous, which is more likely to occur, but has been neglected by most of the current studies.
- Both the erroneous spectrum sensing and the asynchronous inter-channel spectrum sensing risks the PU suffering more interference, which may render the PU unable to work in the interference-sensitive applications. The associated interference reduction problem is addressed, which is of great value.
- The complexity of the proposed method is derived in terms of the average FH number (which indicates the average number of FH should be taken to deliver an SU packet). Thus future applications would be benefited from it.

The rest of the paper is organized as follows. The system model and preliminaries are presented in Section 2. The proposed method is formulated in Section 3. The complexity analysis is provided in Section 4. The simulation results are provided in Section 5. Finally, Section 6 concludes the paper.

## 2. System Model

Asynchronous inter-channel spectrum sensing is universal [19]. A centralized cognitive radio network is considered. Each SU monitors one different PU channel at a certain frequency band, and reports its outcome to the central node for candidate channel selection. For simplicity, we consider that only one SU intends to transmit data to the central node. Our proposed method can be extended to the multiuser case by using distribution coordination [21]. The FH time overhead is considered negligible. Proactive periodical spectrum sensing is employed as a demonstration of the proposed method. Since PU channels are usually in different conditions, the spectrum sensing interval is different from one PU channel to another in order to efficiently discover spectrum opportunities the efficient discovery of spectrum opportunities is falling out of the range of this paper. Thus, the inter-channel spectrum sensing moments are considered asynchronous.

Moreover, regarding cooperative spectrum sensing, usually the reporting time of the spectrum sensing outcome of each cooperator is asynchronous with each other [17][18]. (e.g., due to channel impairments, or observation time offsets). If several PU channels should be sensed cooperatively, in the fusion centers the indicating time of the spectrum sensing outcome of these channels would be asynchronous as well. (For simplicity, cooperative spectrum sensing is not considered in this paper.)

We consider the scenario in which when interference occurs, the SU signal is not disturbed by the PU signal, but the PU signal is seriously interfered by the SU signal, e.g., the SUs are located at the edge of the PU network (the PU signal is relatively weak, while the SU signal is strong), as indicated by **Fig. 2**. Thus, the SU signal would not be corrupted by the interference of the PU signal, and no retransmission scheme is employed.

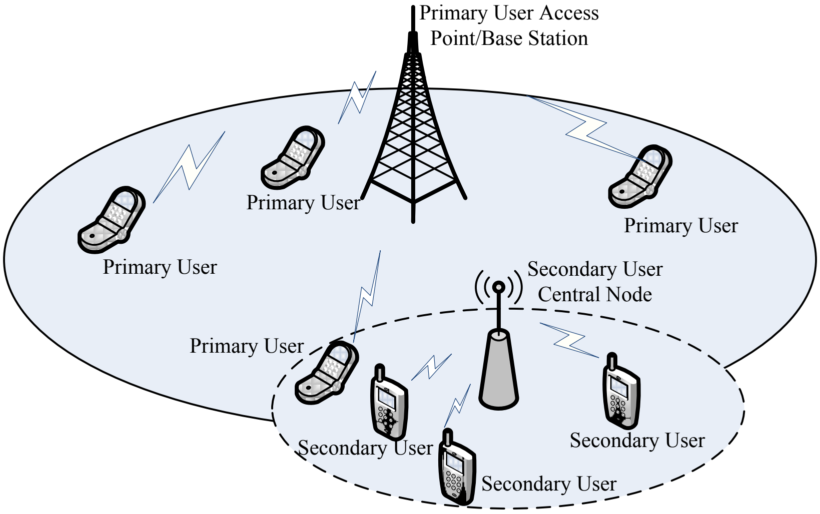

**Fig. 2.** A PU and SU co-existence network

## 2.1 Preliminaries on PU Channel Description

Usually, the state of PU channel *k*, which is observed by a secondary user, is indicated by a binary random variable $Z_k(t)$. $Z_k(t)$ represents the state of channel $k$ at time $t$ with $k \in \{1,2,\dots,N\}$ [13][14][15][16][17][18][19][20].

$$\begin{cases} Z_k(t) = 1 \text{ if channel } k \text{ is busy at time } t \\ Z_k(t) = 0 \text{ otherwise} \end{cases} \tag{1}$$

$Z_k(t)$ follows a continuous-time alternating renewal process [20], which appropriately describes the alternating of PU state between busy/on (state “1”) and idle/off (state “0”). $Z_k(t)$ is characterized by two probability density functions $f_k^1(t)$ and $f_k^0(t)$, which separately denote the probability distribution of the sojourn duration of the busy period $T_k^1(t)$ and idle period $T_k^0(t)$. The on/off periods of PU channel $k$ are considered following exponential distributions [13] as shown by (2), with known $\theta_k^1$ and $\theta_k^0$. $E\{T_k^1(t)\} = 1/\theta_k^1$ (the expectation of $T_k^1(t)$) and $E\{T_k^0(t)\} = 1/\theta_k^0$.

$$\begin{cases} f_k^1(t) = \theta_k^1 e^{\theta_k^1 t} \\ f_k^0(t) = \theta_k^0 e^{\theta_k^0 t} \end{cases} \tag{2}$$

$Z_k(t)$ is described by a set of conditional probabilities in (3),

$$P_k^{i,j}(\Delta t) = u_k^j (1-u_k)^{1-j} + (-1)^{j+i} u_k^{1-i} (1-u_k)^i e^{-(\theta_k^1+\theta_k^0)\Delta t} \tag{3}$$

where $i, j \in \{0,\ 1\}$, $u_k = \theta_k^0 / (\theta_k^0 + \theta_k^1)$ and $\Delta t > 0$. $P_k^{i,j}(\Delta t)$ denotes the probability that state $j$ is now observed, if state $i$ was observed $\Delta t$ time before. The detailed derivation of (3) can be found in [13].

## 2.2 Increased Interference

Due to erroneous spectrum sensing and asynchronous inter-channel spectrum sensing, the interference suffered by PU is increased. The erroneous spectrum sensing would mislead a SU to access a PU channel, which is seemingly “off” but is actually “on”. This results in more interference compared to error-free spectrum sensing.

In the case of asynchronous inter-channel spectrum sensing, regarding the same PU channel conditions, passive DSA would risk the PU suffering more interference compared to synchronous DSA, which can be illustrated by the average interference time. Moreover, since the spectrum sensing is usually erroneous, erroneous spectrum sensing should be considered.

Let us consider channel $k$ is correctly detected as “off” with probability $P_k^d$. Let $T$ denote the time duration of the SU data packet; $t_l^{sub}$ with $l \in \{1, 2, ..., L\}$ denotes the time duration of sub-packet $l$, where $L$ is the total number of sub-packets, and $T = \sum_{l=1}^{L} t_l^{sub}$. Each sub-packet $l$ is continuously delivered by one PU channel, and each sub-packet $l$ is considered consisting of small packets with equal length $dt$ for interference analysis, which can be ignored in application.

For synchronous DSA, the average interference time for delivering a small packet with length $dt$ on channel $k$ can be written as,

$$\beta_k = \frac{\int_0^{I_k} P_k^d P_k^{0,1}(\Delta t) d(\Delta t)}{I_k} dt, \tag{4}$$

where $I_k$ is the spectrum sensing interval of channel $k$. Accordingly, the corresponding average interference time for passive DSA can be expressed as,

$$\beta_{pa(k)} = \frac{\int_{\hat{t}_{as}}^{I_k} P_k^d P_k^{0,1}(\Delta t)d(\Delta t)}{I_k - \hat{t}_{as}} dt \,, \tag{5}$$

where $\hat{t}_{as} = E\{t_{as}\}$, and $t_{as}$ is the asynchronous time relative to the latest spectrum sensing of channel *k* as shown in **Fig. 3**. Since $P_k^{0,1}(\Delta t)$ is monotonically increasing with $\Delta t$, $\beta_{pa(k)} > \beta_k$. (When a small packet is delivered, its channel accessing probability of channel *k*, is considered the same in both synchronous DSA and passive DSA.)

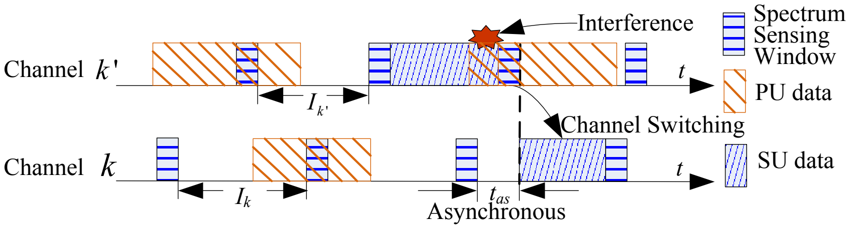

**Fig. 3.** A case of channel switching of passive DSA

## 3. Formulation of Proactive DSA

We formulate the proactive DSA method in this section, and provide a recursive solution by induction.

### 3.1 Problem Formulation of Proactive DSA

To reduce interference, we propose that FH is proactively triggered by the emergence of another desired channel with a lower probability of PU's existence compared to that of the current channel; thus we term it as proactive DSA. If PU's return to the current channel is reported by spectrum sensing before the emergence of this desired channel, SU will stop its data transmision on the current chanel as usual (before this spectrum sensing window) and resume its transmission on the desired channel at the emergence of the desired channel.

Let us first consider the average interference time induced by delivering a small packet *dt* in sub-packet *l*. Define $P_k^f$ and $P_k^d$ as the false alarm probability and the detection probability of the spectrum sensing in channel *k* respectively. $P_k^f$ and $P_k^d$ are considered as known. Channel *K*[*l*] ($K[l] \in \{1,2,...,N\}$, $l \in \{1,2,...,L\}$) is correctly detected as "off" with probability $P_{K[l]}^d$. In this case, if SU data is being delivered by channel *K*[*l*] and PU suddenly returns to channel *K*[*l*], interference would occur. According to (3), if a small packet with length *dt* is delivered by channel *K*[*l*] at time *t*, the average interference time is $P_{K[l]}^{0,1}(t - t_{K[l]}^s)dt$ (i.e., the expected time of channel *K*[*l*] is "on" during *dt*), where $P_{K[l]}^{0,1}(t - t_{K[l]}^s)$ is considered a constant during *dt*, $t_{K[l]}$ denotes the beginning instant of sub-packet *l* accessing channel *K*[*l*], $t_{K[l]}^s$ is the completing instant of the latest spectrum sensing of channel *K*[*l*] before $t_{K[l]}$. Then, the average interference time for delivering all the small packets within sub-packet *l* is the sum of $P_{K[l]}^{0,1}(t-t_{K[l]}^{s})dt$s during $t_l^{sub}$, which can be written as

$$\int_{t_{K[l]}}^{t_{K[l]}+t_l^{sub}} P_{K[l]}^{0,1}(t-t_{K[l]}^{s})dt\,. \tag{6}$$

On the other hand, there is another case in which the interference occurs: channel *K*[*l*] is falsely detected as "off" with probability $P_{K[l]}^{f}$, and SU data is delivered on channel *K*[*l*]. Following the analysis that channel *K*[*l*] is correctly detected as "off", in this case, the average interference time for delivering all the small packets within sub-packet *l* is the sum of $P_{K[l]}^{1,1}(t-t_{K[l]}^{s})dt$s during $t_l^{sub}$, which can be written as,

$$\int_{t_{K[l]}}^{t_{K[l]}+t_l^{sub}} P_{K[l]}^{1,1}(t-t_{K[l]}^{s})dt\,. \tag{7}$$

Thus, regarding erroneous spectrum sensing and asynchronous inter-channel spectrum sensing, the average interference time induced by delivering sub-packet *l* on channel *K*[*l*] can be written as,

$$\tilde{I}_{K[l]}(t_{K[l]},t_l^{sub}) = P_{K[l]}^{d}\int_{t_{K[l]}}^{t_{K[l]}+t_l^{sub}} P_{K[l]}^{0,1}(t-t_{K[l]}^{s})dt+P_{K[l]}^{f}\int_{t_{K[l]}}^{t_{K[l]}+t_l^{sub}} P_{K[l]}^{1,1}(t-t_{K[l]}^{s})dt\,. \tag{8}$$

Denote by *γ*, a FH policy given by sequence $\{t_{K[1]},\ldots, t_{K[l]},\ldots, t_{K[L]}\}$ (hop to channel *K*[*l*] to deliver sub-packet *l* at time $t_{K[l]}$). Therefore, the optimal FH policy $\tilde{\gamma}$ can be obtained from,

$$\tilde{\gamma} = \arg\min_{\gamma\in\Gamma}\{\tilde{I}(\gamma)\} = \arg\min_{\gamma\in\Gamma}\left(\sum_{l=1}^{L}\tilde{I}_{K[l]}(t_{K[l]},t_l^{sub})\right), \tag{9}$$

where the set of admissible policies $\Gamma=\{t_{K[l]}\,|\,t_{K[1]}=t_0<\ldots<t_{K[l]}<\ldots<t_{K[L]}\}$, $t_0$ is a pre-set time.

**Remark 1**: According to (3), $\tilde{I}_{K[l]}(t_{K[l]},t_l^{sub})$ is an integral, which can be solved. It does not depend on *dt*, but depends on $t_{K[l]}$ and $t_l^{sub}$ instead. Thus, the assumption that each sub-packet *l* is considered as consisting of small packets with equal length *dt*, can be ignored.

**Remark 2**: We consider erroneous spectrum sensing (with detection probability $P_k^d$ and false alarm probability $P_k^f$ for channel *k*); therefore if channel *k* is indicated as "off" by the latest spectrum sensing at instant $t_k^s$, the probability of PU's existence in channel *k* at instant *t* ($t_k^s<t<I_k+t_k^s$), can be written as (10). Recall that $t_k^s$ is the completing instant of the latest spectrum sensing of channel *k* and $I_k$ is the spectrum sensing interval of channel *k*.

$$P_k^{PU}(t-t_k^s)=P_k^d\cdot P_k^{0,1}(t-t_k^s)+P_k^f\cdot P_k^{1,1}(t-t_k^s) \tag{10}$$

According to (3), $P_k^{0,1}(t-t_k^s)$ is monotonically increasing with $t$, while $P_k^{1,1}(t-t_k^s)$ is monotonically decreasing with $t$. Usually $P_k^d >> P_k^f$ with $k \in \{1,2,...,N\}$ ; thus (10) is considered monotonically increasing with $t$. We will provide a recursive solution for (9) based on (10) in the next subsection.

### 3.2 Recursive Solution

Since (9) is the minimization of a summation, (9) can be considered as the minimization of a summing process with each sub-summation minimized. That is the spirit of dynamic programming. Thus, (9) can be written as following set of dynamic programming equations, and then recursively solved.

$$U(1) = \min_{K[1]\in\{1,2,...,N\}} \{\tilde{P}_{K[1]}(t_{K[1]}, t_1^{sub})\}, \forall 0 < t_1^{sub} < T \tag{11}$$

$$U(l) = \min_{K[l]\in\{1,2,...,N\}} \{\tilde{P}_{K[l]}(t_{K[l]}, t_l^{sub}) + U(l-1)\}, \tag{12}$$

where $l \in \{2,3,\ldots, L\}$ and $0 < t_l^{sub} < T - \sum_{d=1}^{l-1} t_d^{sub}$ , the equal sign hold as $l$=$L$. (if $t_1^{sub}$=$T$ , L=1 )

$$\min_{\gamma\in\Gamma}\{\tilde{P}(\gamma)\} = \min_{K[l]\in\{1,2,...,N\}}\{U(L)\}. \tag{13}$$

In particular, from (11), (12) and (13), $\tilde{\gamma} = \{t_{K[1]}^* < t_{K[2]}^* < ... < t_{K[L]}^*\}$ can be determined by induction on $l$, which is explained with $N$ = 2 (based on **Fig. 4**) for easy understanding. Spectrum sensing window $t_{win}$ of CH1 and CH2 are considered as the same.

*Induction basis*: $l$=1. $t_{K[1]}^* = t_0$ . $t_0$ is a pre-set time when SU begins to load sub-packet 1 on channel $K$[1], where $K$[1] can be obtained from (14).

$$K[1] = \arg\min_{k\in\{1,2,...,N\}} \{P_k^d \cdot P_k^{0,1}(t-t_k^s) + P_k^f \cdot P_k^{1,1}(t-t_k^s)\} \tag{14}$$

*Induction step*: $l \in \{2,3,\ldots\}$. Without loss of generality, consider sub-packet $l-1$ is being delivered on channel CH1 ( $K[l-1]$= CH1), and $t_{K[l-1]}^*$ is known, while $t_{l-1}^{sub}$ and $t_{K[l]}^*$ are unknown and to be determined. CH2, denoted by $k'$, is latestly reported "off" by spectrum sensing at $t_{k'}^s$ . We define the concerned "off" time of channel $K[l-1]$ as $t_{K[l-1]}^* < t < t_{K[l-1]}^s + I_{K[l-1]}$ , i.e., currently sub-packet $l-1$ is being delivered within this concerned "off" time. We define the concerned "off" time of channel $k'$ as $t_{k'}^s < t < t_{k'}^s + I_{k'}$, i.e., sub-packet $l$ may be delivered within this concerned "off" time. Let $C$=$\{t|t_{K[l-1]}^* < t < t_{K[l-1]}^s + I_{K[l-1]} \& t_{k'}^s < t < t_{k'}^s + I_{k'}\}$ .

The case $C \neq \phi$ ( $\phi$ is an empty set), which means the concerned "off" time of channel $K[l-1]$ and $k'$ overlaps. As $t$ goes on, if the probability of PU's existence in channel $k'$ becomes less than that of channel $K[l-1]$ for $t \in C$ (i.e., (15) becomes less than (16), $t \in C$ ), hop to CH2 ($K[l]{=}k'$) at $t^{*}_{K[l]}{=}t_{k'}$, where at time $t_{k'}$, (15) equals (16), as shown in **Fig. 4**. The length of sub-packet $l-1$ is $t^{sub}_{l-1}{=}t^{*}_{K[l]}-t^{*}_{K[l-1]}$

$$P^{PU}_{k'}(t-t^{s}_{k'})=P^{d}_{k'}\cdot P^{0,1}_{k'}(t-t^{s}_{k'})+P^{f}_{k'}\cdot P^{1,1}_{k'}(t-t^{s}_{k'}) \tag{15}$$

$$P^{PU}_{K[l-1]}(t-t^{s}_{k'})=P^{d}_{K[l-1]}\cdot P^{0,1}_{K[l-1]}(t-t^{s}_{K[l-1]})+P^{f}_{K[l-1]}\cdot P^{1,1}_{K[l-1]}(t-t^{s}_{K[l-1]}) \tag{16}$$

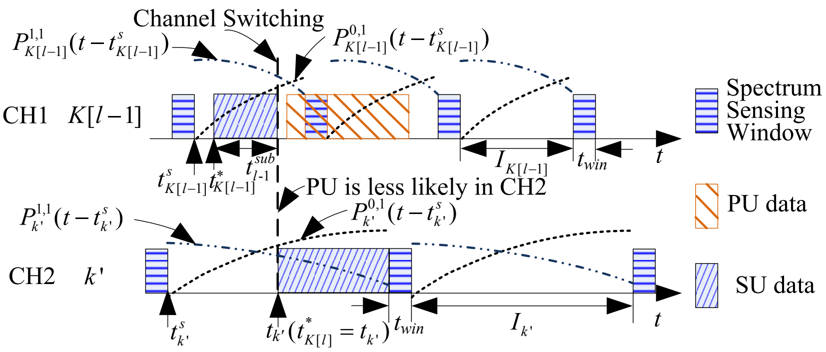

**Fig. 4.** A case of channel switching in proactive DSA with two channels. Because we tackle the erroneous spectrum sensing, the true state of the PU channel cannot be known exactly. Both $P^{0,1}_{k}(\cdot)$ and $P^{1,1}_{k}(\cdot)$ are indicated in this figure.

Otherwise, stay CH1 ($K[l]{=}K[l-1]$) and deliver sub-packet $l$ at $t^{*}_{K[l]}{=}t^{s}_{K[l-1]}+I_{K[l-1]}+t_{win}$ for (*i*) or at $t^{*}_{K[l]}{=}t^{s}_{k'}+I_{k'}+t_{win}$ for (*ii*), if either (*i*) or (*ii*) occurs; switch to CH2 ($K[l]=k'$) at time $t^{*}_{K[l]}{=}t^{s}_{K[l-1]}+I_{K[l-1]}+t_{win}$ for (*iii*) or at time $t^{*}_{K[l]}{=}t^{s}_{k'}+I_{k'}+t_{win}$ for (*iv*), if either (*iii*) or (*iv*) occurs.

- *(i)* $t^{s}_{K[l-1]}+I_{K[l-1]}<t^{s}_{k'}+I_{k'}$ and CH1 is first reported "off" by the very next spectrum sensing, as shown in **Fig. 5**.
- *(ii)* $t^{s}_{K[l-1]}+I_{K[l-1]}\geq t^{s}_{k'}+I_{k'}$ and CH2 is first reported "on" by the very next spectrum sensing.
- *(iii)* if CH1 is first reported "on" in (*i*), rather than "off".
- *(iv)* if CH2 is first reported "off" in (*ii*), not "on".

Among (*i*), (*ii*), (*iii*) and (*iv*), if either (*i*) or (*iii*) occurs, the spectrum sensing window length $t_{win}$ should be subtracted when the value of $t^{sub}_{l-1}$ is calculated, i.e., $t^{sub}_{l-1}=t^{*}_{K[l]}-t^{*}_{K[l-1]}-t_{win}$ otherwise, if either (*ii*) or (*iv*) occurs, $t^{sub}_{l-1}=t^{*}_{K[l]}-t^{*}_{K[l-1]}$.

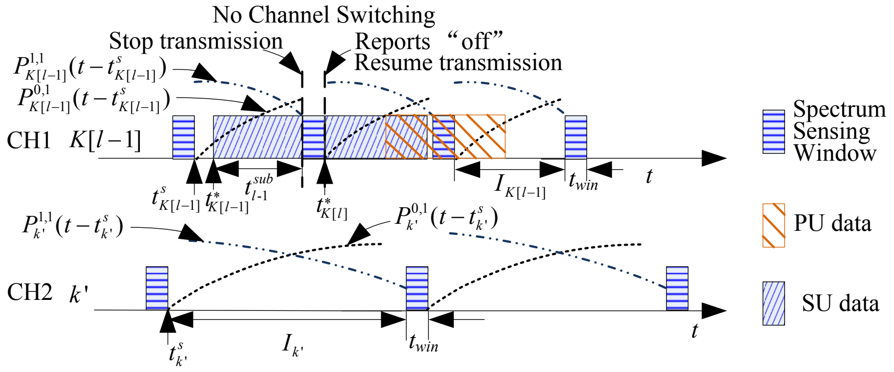

**Fig. 5.** The channel switching in proactive DSA with two channels. The scenario of case (*i*), i.e., $t^s_{K[l-1]}+I_{K[l-1]}<t^s_{k'}+I_{k'}$ and CH1 is first reported "off" by the very next spectrum sensing. The other cases (*ii*), (*iii*) and (*iv*) can be conceived accordingly.

*2)* The case $C=\phi$. which means the concerned "off" time of channel $K[l-1]$ and $k'$ does not overlap. In this case, set $t^{sub}_{l-1}=t^s_{K[l-1]}+I_{K[l-1]}-t^*_{K[l-1]}$, and deliver sub-packet $l$ at $t^*_{K[l]}$ ($t^*_{K[l]}>t^*_{K[l-1]}$) when channel $K[l]$ is the *first* reported "off" among all the PU channels.

While $t^{sub}_{l-1}<T-\sum_{d=1}^{l-2}t^{sub}_d$, one can go on with this procedure to obtain $t^{sub}_l$; otherwise, set $L=l-1$, $t^{sub}_L=T-\sum_{d=1}^{l-2}t^{sub}_d$ and complete the delivery of this SU packet *T*. We summarize above procedures into the flow chart in **Fig. 6**.

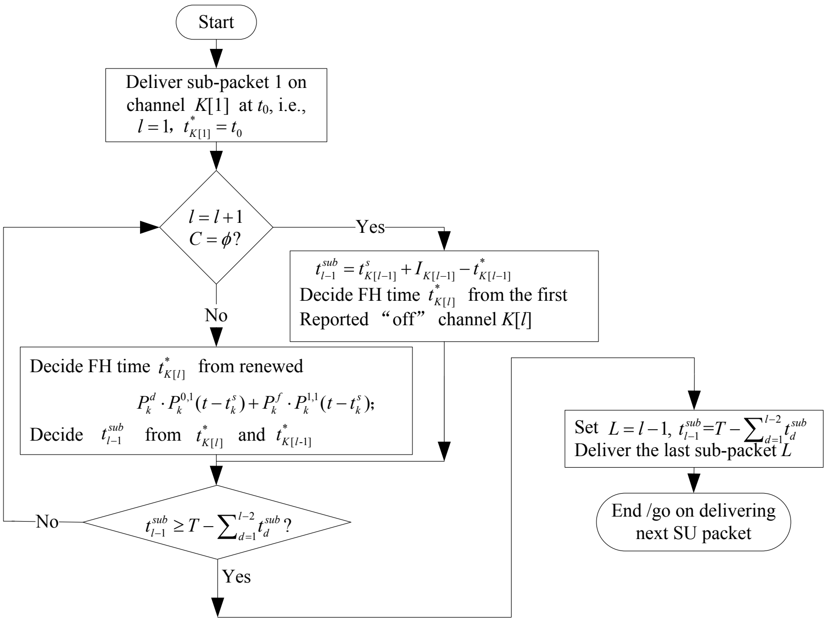

**Fig. 6.** The flow chart of delivering an SU packet *T* by two channels with proactive DSA

**Remark 3**: Once the channel parameters and the current spectrum sensing outcome of channel *k* are known, $P_k^{i,j}(\Delta t)$ with $0<\Delta t<I_k$ can be calculated. That is to say, at the time when the spectrum sensing outcome is indicated, the central node of the SU network is capable of calculating the probability of PU's existence (10) of channel *k* from the value of $P_k^{i,j}(\Delta t)$ for the upcoming time of a spectrum sensing interval $I_k$. That is very helpful for determining $t^*_{K[l]}$, if $t^*_{K[l-1]}$ is known.

## 4. Complexity

The DSA is considered deploying an FH scheme. The algorithm complexity is pertinently related to the average FH number. Thus, the complexity is discussed in terms of it.

Firstly, let us investigate the average FH number of delivering an SU packet *T* with proactive DSA. To obtain the average FH number, we should know the average length of the sub-packet delivered by channel *k*. We denote this average length of sub-packet by $T_k^p$. To calculate $T_k^p$, the probability that channel *k* is preferred over all the other PU channels should be investigated.

Let us consider the probability that channel *k* is preferred over another channel $k'$, which is denoted by $P_k^p(t\,|\,k')$, i.e., the probability of $P_k^{PU}(t-t_k^s)<P_{k'}^{PU}(t-t_{k'}^s)$. Recall $u_k=\theta_k^0/(\theta_k^0+\theta_k^1)$, which is the channel utilization of channel *k*. Correspondingly, we define the channel underutilization of channel *k* as $\alpha_k{=}1-u_k$. $\alpha_k$ and $u_k$ indicate the true probabilities of channel *k* being "off" and "on", respectively. Thus, the probability of channel *k* being "off" discovered by spectrum sensing is

$$\alpha_k\cdot P_k^d+u_k\cdot P_k^f\,. \tag{17}$$

Channel *k* being detected as "off" is a premise for the probability that channel *k* is preferred over channel $k'$; therefore there is no need of indicating again the probability of channel *k* being detected as "off" in the expression of $P_k^p(t\,|\,k')$, or the probability of channel *k* being detected as "off" can be considered integrated into $p_k^{acc}$ which denotes the channel access probability of channel *k*: now that channel *k* is accessible, it should be detected as "off". Thus, $P_k^p(t\,|\,k')$ can be written as,

$$P_k^p(t\,|\,k'){=}\begin{cases}1-(\alpha_{k'}\cdot P_{k'}^d+u_{k'}\cdot P_{k'}^f)+(\alpha_{k'}\cdot P_{k'}^d+u_{k'}\cdot P_{k'}^f)\dfrac{I_{k'}-\Delta t_{k'}}{I_{k'}{+}t_{win}} & \text{if } I_{k'}-\Delta t_{k'}>0\\[2ex] 1-(\alpha_{k'}\cdot P_{k'}^d+u_{k'}\cdot P_{k'}^f) & \text{otherwise}\end{cases} \tag{18}$$

where $\Delta t_{k'}{=}P_{k'}^{PU}(P_k^{PU}(\Delta t_k))^{-1}$, $P_{k'}^{PU}(\cdot)^{-1}$ is the inverse function of $P_{k'}^{PU}(\cdot)$, $\Delta t_k{=}t-t_k^s$ and $\Delta t_{k'}{=}t-t_{k'}^s$. The derivation of $P_k^p(t\,|\,k')$ is in Appendix.

Let $P_k^{min\_p}(\Delta t_k)$ denote the probability that channel *k* is preferred over all the other $N-1$ channels. Thus, it can be written as,

$$P_k^{min\_p}(\Delta t_k)=\prod\nolimits_{k'=1,k'\neq k}^{N} P_k^{p}(t\,|\,k') \tag{19}$$

Since FH is facing asynchronous inter-channel spectrum sensing, the length of the sub-packet delivered at channel $k$ corresponding to probability $P_k^{min\_p}(\Delta t_k)$, can be any variable within $(0,\ \Delta t_k]$. We consider this sub-packet length is randomly distributed within $(0,\ \Delta t_k]$. The expectation $\frac{\Delta t_k}{2}$ is selected, thus $T_k^p$ can be written as,

$$T_k^p=\int_0^{I_k} P_k^{min\_p}(\Delta t_k)\frac{\Delta t_k}{2}d(\Delta t_k) \tag{20}$$

Thus, the average FH number of delivering an SU packet $T$ with proactive DSA can be written as

$$\sum\nolimits_{k=1}^{N} p_k^{acc}\frac{T}{T_k^p} \tag{21}$$

Accordingly, the average FH number of delivering an SU packet $T$ with synchronous FH-DSA can be written as

$$\sum\nolimits_{k=1}^{N} p_k^{acc}\frac{T}{I_k} \tag{22}$$

For passive DSA, due to asynchronicity the sub-packet length is randomly distributed within $(0,\ I_k]$. The average length of the sub-packet delivered by channel $k$ is considered as the expectation of $(0,\ I_k]$, i.e., $\frac{I_k}{2}$. Therefore, the average FH number of delivering an SU packet $T$ with passive DSA can be written as

$$2\sum\nolimits_{k=1}^{N} p_k^{acc}\frac{T}{I_k} \tag{23}$$

When a small packet with length $dt$ is delivered, its channel accessing probability of channel $k$, is considered the same for synchronous DSA, passive DSA, and the proposed method.

In [16], a proactive planning policy is proposed, i.e., switch to another desired channel if with high probability that the length of the remaining idle period of this desired channel is larger than that of the current channel in use by SU. We term it as proactive policy. It has outperformed the other policies proposed by [16]. Due to the channel access manner of the proactive policy, the expression of its average FH number is difficult to find. In the case of asynchronous inter-channel spectrum sensing, the average FH number of the proactive policy is greater than that of synchronous DSA (22); but less than that of passive DSA (23), because of smart channel switch as claimed in [16].

It is cumbersome to directly compare the FH numbers of aforementioned methods as usual. We will test it by simulations in next section.

## 5. Experimental Results and Analysis

In this section, the proposed method is compared with synchronous DSA, passive DSA and the proactive policy in [16] in terms of interference reduction and complexity.

### 5.1 Interference Reduction

In **Fig. 7**, we plot the average interference time of different methods versus $E\{T_k^1\}/E\{T_k^0\}$, which is obtained from 1000 separate runs. Some of following parameters are set from [13]. $T$ is selected as 10s. For simplicity, $N = 2$, both channels are with the same spectrum sensing window $t_{win}$=0.01s and the same $E\{T_k^1\}/E\{T_k^0\}$ value. For channel 1, fix $E\{T_1^1\}$=1s, $I_1$=0.05s; for channel 2, fix $E\{T_2^1\}$=0.5s, $I_2$=0.06s. $T_{pr}$, $T_{pa}$, $T_{syn}$, and $T_{pp}$, denote the average interference time induced by delivering an SU packet $T$ with proactive DSA, passive DSA, synchronous DSA and the proactive policy in [16] respectively. As a demonstration of erroneous spectrum sensing, $P_k^d$ is set to 0.95 for all the channels and $P_k^f = 0.05$.

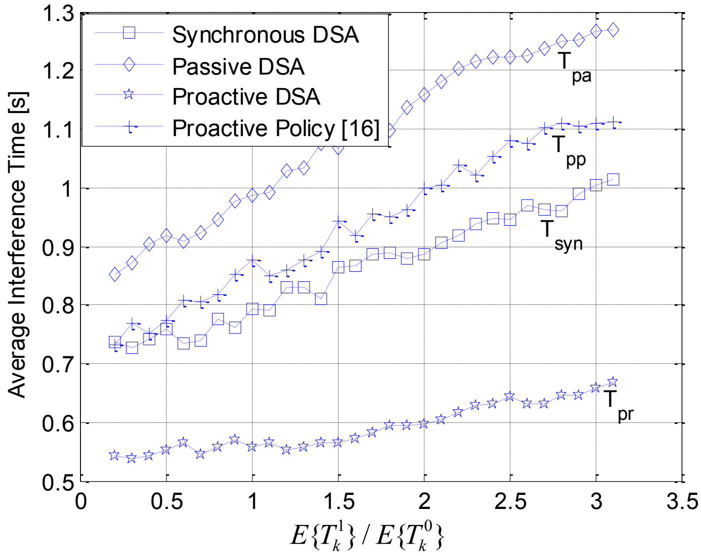

**Fig. 7.** The average interference time of delivering an SU packet $T$ by two channels with various DSA methods

As expected, in **Fig. 7**, $T_{pr} < T_{pa}$ regarding same $E\{T_k^1\}/E\{T_k^0\}$ value. $T_{pa} > T_{syn}$ is committed by asynchronous spectrum sensing. Since proactive DSA always hops to a channel with lower probability of PU's existence, this policy renders its average interference less than $T_{syn}$, i.e., $T_{pr} < T_{syn}$ , as shown in **Fig. 7**. The average interference induced by the proactive policy in [16] is less than $T_{pa}$ which is because the proactive policy has made smart channel switches as claimed in [16]; and greater than $T_{syn}$ which is due to asynchronous inter-channel spectrum sensing.

### 5.2 Complexity

We have tested the complexity in terms of the average FH number for delivering an SU packet $T$ with various DSA methods. $p_k^{acc}$ is considered proportional to the channel underutilization $\alpha_k$, i.e., $p_k^{acc} = \frac{\alpha_k}{\sum_{d=1}^{N} \alpha_d}$. The parameter settings are the same as in section 5.1.

In **Fig. 8**, we plot the average FH number of different methods versus $E\{T_k^1\}/E\{T_k^0\}$ values, which is also obtained from 1000 separate runs. As expected, regarding same $E\{T_k^1\}/E\{T_k^0\}$ value, the tested value of the average FH number of passive DSA (denoted by passive DSA in **Fig. 8**) is about two times that of synchronous DSA (denoted by synchronous DSA in **Fig. 8**). The tested values of average FH number of synchronous DSA and passive DSA are a bit less than their theoretical values respectively. We attribute this phenomenon to that: the channel access probaiblity of synchronous DSA and passive DSA does not exactly follow $p_k^{acc}$, due to their respective channel access manners.

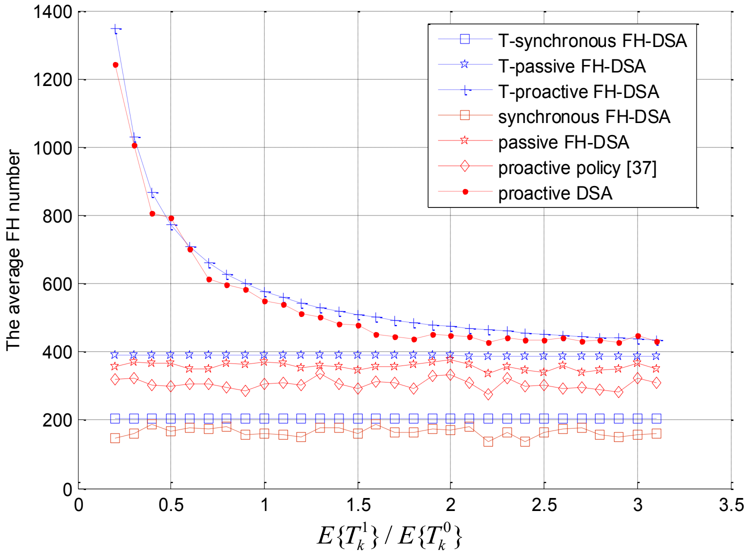

**Fig. 8.** The average FH number of delivering an SU packet $T$ by two channels with various DSA methods. "T-synchronous DSA" indicates the theoretical value of the average FH number of the synchronous DSA (T stands for "theoretical", which are the same for passive DSA and proactive DSA)

As shown in **Fig. 8**, the theoretical value of the average FH number of proactive DSA, is greater than those of the other methods, when $E\{T_k^1\}/E\{T_k^0\}$ is small, and it decreases with $E\{T_k^1\}/E\{T_k^0\}$. As the $E\{T_k^1\}/E\{T_k^0\}$ value increases, the "off" duration of channel $k$ becomes less ($k \in \{1,2,\ldots,N\}$). According to proactive DSA with a larger $E\{T_k^1\}/E\{T_k^0\}$ value, FH is frequently carried out in the manner of case 2) ($C = \phi$) in the induction step $l$ in Section 3.2 or in the manner of the corresponding case 1) with a relatively bigger sub-packet size. Therefore the average FH number becomes less. Consider the extreme scenario: all FH is carried out in the manner of case 2) in the induction step $l$ in Section 3.2; or that the length of the sub-packets delivered in the manner of the corresponding case 1) is almost as large as the spectrum sensing interval. Therefore the average FH number of proactive DSA approaches that of passive DSA as $E\{T_k^1\}/E\{T_k^0\}$ increases.

The tested value of the average FH number of proactive DSA, which is indicated by "proactive DSA", follows its theoretical value. As expected, the tested values of the proactive policy in [16], are distributed between those of synchronous DSA and those of passive DSA in **Fig. 8**.

## 6. Conclusion

The proactive DSA has efficiently reduced interference against both erroneous spectrum sensing and asynchronous inter-channel spectrum sensing. In particular, the erroneous spectrum sensing is accounted for by a detection probabiliy, while proactive FH is conceived against the asynchronous inter-channel spectrum sensing. In the proposed method, FH is proactively triggered by the emergence of another channel with a lower probabability of PU's existence compared to that of the current channel in use by SU.

Furthermore, the application of the idea of proactive FH is not limited to this paper. It is also applicable to other FH based communication systems to reduce interference, e.g., the scenarios in [16]. The FH time overhead is deemed negligible and SU throughput is not concerned. Their further study can be our future work.

## Appendix the Derivation of $P_k^p(t\,|\,k')$

$P_k^p(t\,|\,k')$, which indicates probability of channel $k$ is preferred over another channel $k'$, can be considered in the following two cases (the probability of channel $k$ being "off" is considered integrated into $p_k^{acc}$)

Case $C1$: channel $k$ is preferred over channel $k'$, only when channel $k'$ is indicated "on" by spectrum sensing;

Case $C2$: channel $k$ is preferred over channel $k'$, when channel $k'$ is indicated "on" and for a certain time when $k'$ is indicated "off" according to spectrum sensing outcome.

In case $C1$, the channel condition of channel $k$ is relatively poorer than that of channel $k'$. Channel $k$ is preferred over channel $k'$, only when channel $k$ is indicated "off" by spectrum sensing while channel $k'$ is indicated "on" Thus, in this case, $P_k^p(t\,|\,k')$ is equal to the probability that channel $k'$ is indicated "on" by spectrum sensing, i.e., $1-(\alpha_{k'}\cdot P_{k'}^d+u_{k'}\cdot P_{k'}^f)$ . Recall that the probability of channel $k'$ is indicated "off" by spectrum sensing is $\alpha_{k'}\cdot P_{k'}^d+u_{k'}\cdot P_{k'}^f$, as expressed by (17).

In case $C2$, channel $k$ is preferred over channel $k'$, for two kinds of time durations: 1) channel $k$ is indicated "on" (the same as that in case $C1$); 2) for a certain time when $k'$ is indicated "off" as shown in **Fig. 9**. Thus, in this case, $P_k^p(t\,|\,k')$ is equal to the probability that channel $k'$ is indicated as "off" during a certain time.

According to **Fig.9**, let $\Delta t_k = t - t_k^s$ and $\Delta t_{k'} = t - t_{k'}^s$ , the occurrence probability of the interested duration can be written as

$$(\alpha_{k'}\cdot P_{k'}^d+u_{k'}\cdot P_{k'}^f)\frac{I_{k'}-\Delta t_{k'}}{I_{k'}+t_{win}}, \tag{24}$$

where $(\alpha_{k'}\cdot P_{k'}^d+u_{k'}\cdot P_{k'}^f)$ is the probability of channel $k'$ indicated "off" by spectrum sensing. $I_{k'}+t_{win}$ is the spectrum sensing interval of channel $k'$ (including spectrum sensing window length $t_{win}$). $I_{k'} - \Delta t_{k'}$ is the length of interested duration.

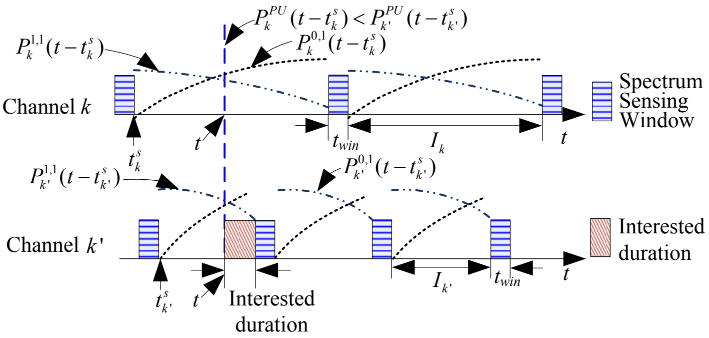

**Fig. 9.** Channel $k$ is preferred over channel $k'$ for a certain time when channel $k'$ is indicated "off" (the interested duration).

Thus, $P_k^p(t\,|\,k')$ can be written as

$$P_k^p(t\,|\,k') = \begin{cases} 1-(\alpha_{k'}\cdot P_{k'}^d + u_{k'}\cdot P_{k'}^f) + (\alpha_{k'}\cdot P_{k'}^d + u_{k'}\cdot P_{k'}^f)\dfrac{I_{k'}-\Delta t_{k'}}{I_{k'}+t_{win}} & \text{if } \Delta t_{k'} < I_{k'} \\ 1-(\alpha_{k'}\cdot P_{k'}^d + u_{k'}\cdot P_{k'}^f) & \text{otherwise} \end{cases}, \quad (25)$$

where $\Delta t_{k'} = P_{k'}^{PU}(P_k^{PU}(\Delta t_k))^{-1}$. $P_{k'}^{PU}(\cdot)^{-1}$ is the inverse function of $P_{k'}^{PU}(\cdot)$.

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