

Resource Allocation based on Hybrid Sharing Mode for Heterogeneous Services of Cognitive Radio OFDM Systems

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Abstract

In cognitive radio networks (CRNs), hybrid overlay and underlay sharing transmission mode is an effective technique for improving the efficiency of radio spectrum. Unlike existing works in the literature, where only one secondary user (SU) uses overlay and underlay modes, the different transmission modes should be allocated to different SUs, according to their different quality of services (QoS), to achieve the maximal efficiency of radio spectrum. However, hybrid sharing mode allocation for heterogeneous services is still a challenge in CRNs. In this paper, we propose a new resource allocation method for hybrid sharing transmission mode of overlay and underlay (HySOU), to achieve more potential resources for SUs to access the spectrum without interfering with the primary users. We formulate the HySOU resource allocation as a mixed-integer programming problem to optimize the total system throughput, satisfying heterogeneous QoS. To decrease the algorithm complexity, we divide the problem into two sub-problems: subchannel allocation and power allocation. Cutset is used to achieve the optimal subchannel allocation, and the optimal power allocation is obtained by Lagrangian dual function decomposition and subgradient algorithm. Simulation results show that the proposed algorithm further improves spectrum utilization with a simultaneous fairness guarantee, and the achieved HySOU diversity gain is a satisfactory improvement.

Keywords: Cognitive radio, Resource allocation, Hybrid sharing mode, Heterogeneous services

1. Introduction

Wireless communication systems are used to deliver all types of heterogeneous and wideband services to mobile users, including voice, message, video, and bitstreams, which cause an extreme scarcity of wireless resources. Cognitive radio (CR) [1,2,3,4] is a promising technique to improve the efficiency of the spectrum. In particular, 3GPP's Long Term Evolution (LTE), based on Orthogonal Frequency-Division Multiplexing (OFDM) with CR, represents an excellent system because of its flexibility in dynamic resource allocation, particularly, by cognizing and handling all subcarriers separately [5]. The research on dynamically allocating the cognitive resources for satisfying heterogeneous QoS services in OFDM cognitive radio networks (CRNs) is still open to further investigation.

Underlay and overlay are two different sharing transmission modes that enable secondary users (SUs) to share the radio spectrum licensed to primary users (PUs) [6], in which SUs are allowed to use the busy or idle subchannels of PUs, respectively. In current literature related to hybrid overlay and underlay sharing transmission modes, most works consider only one SU, using overlay and underlay modes, without considering the heterogeneous services. In [7,8], joint overlay and underlay power allocation for CRNs is studied, whereby the total capacity is maximized while maintaining a total power budget, and keeping the interference introduced to the PU below a threshold for only one SU, is proposed in [7]. The power allocation problem for relay-assisted secondary transmissions in a hybrid overlay and underlay spectrum sharing CRN, where the SUs join the power auction organized by the relay and bid for maximizing the utility, is studied in [8]. In [9, 10], switches between overlay and underlay sharing transmission modes are proposed. The system occasionally switches to an underlay CR mode in [9], although it generally operates in an overlay CR mode to maximize throughput of the SU while satisfying the target departure rate of the PU and securing stability of the SU's transmit queue. In [10], the switch between overlay and underlay sharing modes for an SU is studied to improve its throughput with limited sensing capability, using Markov chains. In [11], an extended, soft decision, spectrally modulated, spectrally encoded framework is studied to generate overlay, underlay, and hybrid overlay and underlay waveforms dynamically in the CR context over frequency selective fading channels to maximize spectrum efficiency and channel capacity.

However, all these works mentioned focus only on one type of service. Heterogeneous services with different QoS are supported in CRN, therefore, **the differences between SUs with different QoS should be utilized to allocate the overlay and underlay subchannels optimally and simultaneously.**

In this paper, we propose a novel resource allocation method of a hybrid sharing transmission mode of overlay and underlay (HySOU) to support heterogeneous services in OFDM CRNs. For real-time (RT) and non-real-time (NRT) users in OFDM CRNs, a secondary base station (sBS) makes a joint allocation of subchannel and power to SUs who have different QoS. The available subchannels are allocated to each of the SUs under the constraints of poverty line and channel state, and the sBS allocates power to the corresponding SUs according to different QoS requirements, without unaccepted interference to the PUs, and then the information for the SUs is transmitted in double sharing modes simultaneously. For successful transmission to RT SUs, a minimum rate is guaranteed. For NRT SUs, a best effort with fairness is adopted by introducing a proportional-fairness constraint. To maximize the system throughput, we formulate the proposed method as a mixed-integer program and solve it

through two sub-problems based on a cutset and Lagrangian dual function decomposition algorithm, called the HySOU algorithm. The HySOU diversity gain is defined as a performance metric to describe the processing effect and compared with each sharing transmission mode.

The remainder of this paper is organized as follows. In Section II, the system model for resource allocation in OFDM CRNs with heterogeneous services is introduced. The joint optimal subchannel and power allocation algorithm is proposed in section III. In Section IV, the simulation results are illustrated. Finally, we conclude this paper in Section V.

2. System Model

OFDM CRN containing a primary network (PN) and a secondary network (SN) is shown in Fig. 1. It is assumed that the PN is an M users OFDM system sharing N subchannels, and the PUs randomly use several subchannels from all these N subchannels at every time slot. The SUs sense all channel states for each time slot and send the sense information to the sBS. The sBS divides all subchannels into two categories, according to the information received from the SUs: idle and busy subchannels (also known as overlay and underlay subchannels), are denoted as N_I and $N_B^m, m \in M$, respectively, where $N_B^m, m \in M$ is the busy subchannels occupied by the m^{th} PU. All idle and busy subchannels are illustrated in Fig. 1.

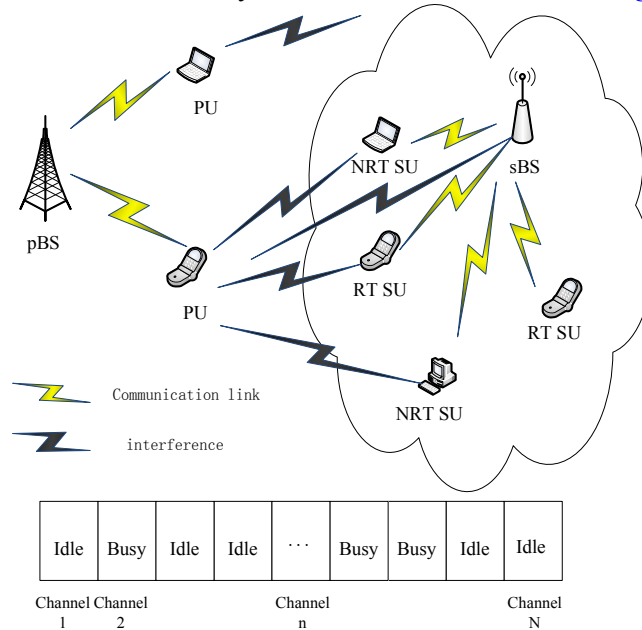


Fig. 1. The OFDM CRNs system scenario and the spectrum licensed to the PN.

Assuming that the cognitive radio system provides all types of heterogeneous services to all users, the SUs are classified into RT and NRT SUs, denoted as K_1 and K_2 , respectively. To improve the utilization of radio spectrum further, we propose a hybrid sharing transmission mode of overlay and underlay (HySOU), in which the overlay and underlay modes are combined simultaneously to allocate the cognitive resources to SUs. To satisfy the different QoS requirements, we give priority to the RT SUs to use the idle subchannels. When the overlay subchannels are insufficient, the NRT SUs reuse the busy subchannels simultaneously with PUs, without causing unaccepted interference to PUs, in underlay mode.

We are interested in CR downlink transmissions when data is transmitted from the sBS to the SUs. The sBS allocates the N subchannels and total power to K SUs, according to the received channel state information of the transmitter (CSI-T).

To describe the performance of the proposed HySOU algorithm, we define a HySOU diversity gain as

$$G = \frac{R_{double}}{R_{single}} \quad (1)$$

where R_{double} is the system throughput with the HySOU, and R_{single} is the system throughput with a single sharing transmission mode [12].

Assuming that the sBS is able to distinguish idle and busy subchannels accurately, the background noise is additive white Gaussian noise (AWGN) with a power spectrum density N_0 , the transmission bandwidth on each subchannel is W , and the interference between the subchannels is neglected. Then the system throughput for the k^{th} SU, **which includes RT and NRT SUs**, is expressed as [13]

$$R_k = \sum_{n=1}^N \rho_{k,n} W \log_2 \left(1 + \frac{1.5 P_{k,n} h_{k,n}^2 / (N_0 W + I_n)}{\ln(0.2 / BER^{tar})} \right) \quad (2)$$

where BER^{tar} is the target bit error rate. For simplicity, each subchannel is assigned to one SU only, and $\rho_{k,n} = 1$ represents that the n^{th} subchannel is allocated to the k^{th} SU, otherwise $\rho_{k,n} = 0$. The transmit power for the k^{th} SU on the n^{th} subchannel is $P_{k,n} \geq 0$, and $h_{k,n}$ is the channel gain from the sBS to the k^{th} SU on the n^{th} subchannel, $I_n = P_{m,n} (f_n^{m,k})^2$ is the interference introduced to the k^{th} SU on the n^{th} underlay subchannel by the m^{th} PU, where $P_{m,n} \geq 0$ is the transmit power of the m^{th} PU on the n^{th} subchannel, $f_n^{m,k}$ is the channel gain from the m^{th} PU to the k^{th} SU on the n^{th} subchannel, and the interference on the overlay subchannels is assumed to be zero.

A minimum rate constraint should be satisfied for RT SUs to guarantee their QoS requirements, so let R_k^{min} be the minimum rate threshold of the k^{th} SU; therefore, the constraint is described as

$$R_k \geq R_k^{min} \forall k \in \mathbf{K}_1 \quad (3)$$

To guarantee the fairness between NRT SUs, we introduce the normalized proportional fairness factor r_k , thus, we have the constraint as

$$R_1 : R_2 : \dots : R_{K_2} = r_1 : r_2 : \dots : r_{K_2} \forall k \in \mathbf{K}_2 \quad (4)$$

where $r_k, \forall k \in \mathbf{K}_2$ is a predetermined value.

In addition, the interference to PUs caused by NRT SUs should be lower than the maximum interference temperature of PUs. The constraint is presented as

$$\sum_{k \in \mathbf{K}_2} \sum_{n=1}^{N_m} \rho_{k,n} o_{m,n}^2 P_{k,n} \leq \delta_m, m \in \mathbf{M} \quad (5)$$

where $o_{m,n}$ is the channel gain from the sBS to the m^{th} PU on the n^{th} subchannel, δ_m is the power threshold of the m^{th} PU, which is the product of the interference temperature, the bandwidth and the Boltzmann constant (1.38J/K) [14].

Our objective is to maximize the secondary system throughput by optimizing the subchannel and power allocation. The optimization problem can be formulated as

$$\begin{aligned}
& \max_{\rho_{k,n}, P_{k,n}} \left(\sum_{k_1=1}^{K_1} R_{k_1} + \sum_{k_2=1}^{K_2} R_{k_2} \right) \\
\text{s.t. } & \sum_{k=1}^K \sum_{n=1}^N \rho_{k,n} P_{k,n} \leq P_{Total} \\
& P_{k,n} \geq 0 \forall k, \forall n, \\
& \rho_{k,n} \in \{0,1\} \forall k, \forall n \\
& \sum_{k=1}^K \rho_{k,n} = 1 \forall n, \sum_{k \in K_2} \sum_{n=1}^{N_2^m} \rho_{k,n} \sigma_{m,n}^2 P_{k,n} \leq \delta_m, m \in \mathbf{M} \quad (6) \\
& PL(k) = \left\lfloor \frac{N_k}{\Delta(k)+1} \right\rfloor \forall k \in \mathbf{K} \\
& R_k \geq R_k^{\min} \forall k \in \mathbf{K}_1 \\
& R_1 : R_2 : \dots : R_{K_2} = r_1 : r_2 : \dots : r_{K_2} \forall k \in \mathbf{K}_2
\end{aligned}$$

where P_{Total} is the total power budget at the sBS, $PL(k)$ is the smallest number of subchannels allocated to the k^{th} SU, which is called the poverty line [15]. N_k is the total number of homogeneous subchannels which the k^{th} SU can use in CRNs, $\Delta(k)$ is the number of SUs similar to the k^{th} SU.

3. Resource Allocation Algorithm

Theoretically, joint subchannel and power allocation is able to achieve the optimal solution of (6). However, the computational complexity is enormous, because discrete and continuous variables exist simultaneously in the mixed-integer programming problem (6). For simplicity, we divide this problem into two parts. First, we propose a novel subchannel allocation scheme, based on cutset. Second, we propose a novel power allocation algorithm, based on the Lagrangian dual function decomposition method [16] and subgradient algorithm.

3.1 Optimal Subchannel Allocation

Inspired by [17], we propose a novel subchannel allocation algorithm. In a [Remark 1] subchannel allocation algorithm, the SU with a number of subchannels below the poverty line has priority to select the subchannel with the highest signal-to-noise ratio (SNR). Each of the subchannels in the same mode has the same power. Then, problem (6) is rewritten as

$$\begin{aligned}
& \max_{\rho_{k,n}} \left\{ \sum_{k=1}^{K_1} \sum_{n=1}^{N_1} \rho_{k,n} W \log_2 \left(1 + \frac{1.5P_1 h_{k,n}^2 / N_0 W}{\ln(0.2/BER^{TAR})} \right) + \sum_{k=1}^{K_2} \sum_{n=1}^{N_2^m} \rho_{k,n} W \log_2 \left(1 + \frac{1.5P_2 h_{k,n}^2 / (N_0 W + I_n)}{\ln(0.2/BER^{TAR})} \right) \right\} \\
\text{s.t. } & \rho_{k,n} \in \{0,1\} \forall k, \forall n \quad (7) \\
& \sum_{k=1}^K \rho_{k,n} = 1 \forall n \\
& PL(k) = \left\lfloor \frac{N_k}{\Delta(k)+1} \right\rfloor \forall k \in \mathbf{K}
\end{aligned}$$

where P_1 and P_2 are predetermined power values for the idle and busy subchannels, respectively. Then, problem (6) is equivalently converted to an integer programming problem (7).

To solve problem (7), cutset [18, 19] is used to achieve the optimal subchannel allocation. The subchannel allocation of CRNs with K_1 RT SUs and N_1 idle subchannels is abstracted as a weighted directed graph $G_1(V_1, A_1, C_1)$, as shown in Fig. 2 (a). V_1 is the vertex set containing the RT SUs and the idle subchannels, A_1 is the edge set from SUs to every idle subchannel, and C_1 is the weighted set of SNR for the k^{th} SU in the n^{th} subchannel, defined as $H_{k,n} = h_{k,n}^2 / N_0 W$. Similarly the subchannel allocation for K_2 NRT SUs and N_2 busy subchannels is abstracted as $G_2(V_2, A_2, C_2)$, and C_2 is the weighted set of SINR for the k^{th} SU in the n^{th} subchannel, defined as $H_{k,n} = h_{k,n}^2 / (N_0 W + I_n)$. With the example of $G_1(V_1, A_1, C_1)$, the detailed subchannel allocation algorithm is described as follows.

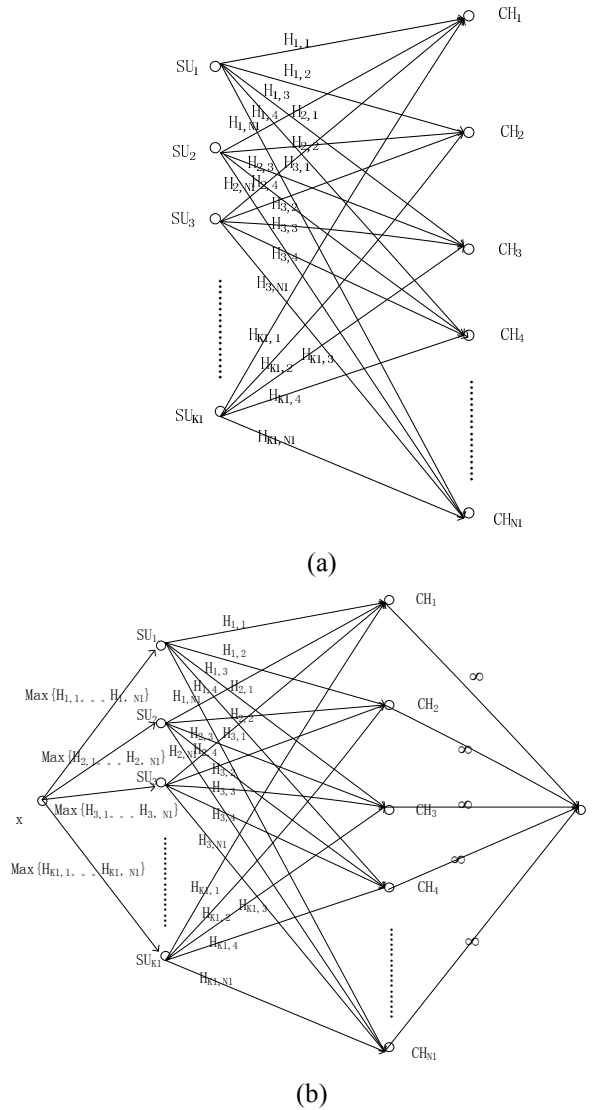


Fig. 2. Weighted directed graph. (a) Multisource and multisink weighted directed graph; (b) Single source and single sink weighted directed graph

We use two steps to obtain the allocation solution. First, the biggest weighted edges are obtained by max-flow min-cut theorem [20]. To exploit max-flow min-cut theorem, the multisource and multisink graph $G_1(V_1, A_1, C_1)$ is converted to a single source and single sink graph $G_1^*(V_1, A_1, C_1)$, as shown in Fig. 2 (b), by adding a virtual source x and a virtual sink y . Then, the maximum flow from x to y is necessary to obtain the biggest weighted edges. To find the maximum flow from x to y , we need to learn the lemmas as follows.

Lemma 1: *Let f be a feasible flow in the network N , P is the direction of x - y , then, the arc (v_i, v_j) in P is called an incrementing arc when it satisfies one of the following two conditions:*

- (1) $(v_i, v_j) \in P^+$, and $f_{ij} < C_{ij}$, then the arc is an unsaturated arc.
- (2) $(v_i, v_j) \in P^-$, and $f_{ij} > 0$, then the arc is a nonempty arc.

If all arcs in P are incrementing arcs, then P is the incrementing path of f .

Lemma 2: *The feasible flow f in the network N is the maximum flow when, and only when, there does not exist an x - y incrementing path of f in the network N .*

We define $G_1^*(V_1, A_1, C_1)$ is the network N , $v_i, v_j \in \{V_1 + \{x, y\}\}$, and f_{ij} , C_{ij} are the flow and capacity from v_i to v_j , respectively. The edge capacity from x to the k^{th} SU, from the k^{th} SU to the n^{th} subchannel and from the n^{th} subchannel to y are $\text{Max}\{H_{k,n}, n=1,2,\dots,N_1\}$, the corresponding edge weighted and ∞ , respectively. P^+ is the direction of x - y , and P^- is the direction of y - x .

Theorem 1: *We define the initial flow from x to the k^{th} SU as the corresponding edge capacity, for every SU, the flow from the SU to the n^{th} ($n=1,2,\dots,N_1$) subchannel with the biggest weighting is the relevant edge capacity, and the flow of others is zero. Then, the initial flow is the maximum flow of the network.*

Proof of theorem 1: It is not necessary to show that no x - y incrementing path for the initial flow in $G_1^*(V_1, A_1, C_1)$ exists, to prove the initial flow is the maximum flow. In $G_1^*(V_1, A_1, C_1)$, we find that the edge capacity from x to every SU is equal to the flow of the edges, according to lemma 1, which means that there does not exist the x - y incrementing path for the initial flow. Then, we prove that the initial flow is the maximum flow of $G_1^*(V_1, A_1, C_1)$, according to lemma 2.

Second, according to the biggest weighted edges which are contained in the maximum flow, the subchannels are allocated to the SUs under the poverty line constraint. Ω_k is the set of subchannels assigned to the k^{th} SU. $\text{count}(k)$ is the number of subchannels in Ω_k . The whole procedure of the subchannel allocation algorithm is described as Table 1.

Table 1. Subchannel allocation algorithm

1: Initialization
2: Set $R_k = 0$ $\text{count}(k) = 0 \quad \forall k \in K_I$;
3: Calculate $PL(k)$, $\forall k \in K_I$.
4: Construct $G_1^*(V_1, A_1, C_1)$ corresponding to the system as described previously.
5: Subchannel Allocation
6: for $k = 1$ to K_I , find n that $ H_{k,n} > H_{k,j} \quad \forall n, j \in N_I$, let the flow from x to the k^{th} SU and from the k^{th} SU to n^{th} subchannel be $ H_{k,n} $, denoted by $f_{x,k}$ and $f_{k,n}$ respectively.
7: end for

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8: if  $f_{k,n} > 0 \cap f_{i,n} = 0, \forall i \in K_I - \{k\}$ 
     $\Omega_k = \Omega_k \cup \{n\}, count(k) = count(k) + 1, N_I = N_I - \{n\}$  and update  $R_k$ .
9: else find  $f_{k,n} > f_{i,n}$  for  $f_{i,n} > 0, \forall i \in K_I$ 
    let  $\Omega_k = \Omega_k \cup \{n\}, count(k) = count(k) + 1, N_I = N_I - \{n\}$  and update  $R_k$ .
10: end if
11: for  $k = 1$  to  $K_I$ , find  $k$  that  $count(k) > PL(k)$ , let  $V_I = V_I - \{k\}$ ,
12: end for
13: if  $V_I \neq \Phi$  go to 6
14: else go to 16
15: end if
16: While  $B_I \neq \Phi$ 
17: Construct  $G_I^*(V_I, A_I, C_I)$ .
11: Do 6,7, 8, 9, and 10.
12: end while

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Theoretically, K_I subchannels can be allocated to SUs when the maximum flow is obtained, then the subchannels allocated to SUs will be removed from V_I , and a new CRNs graph will be formed to continue the allocation process until the end of the subchannel allocation. So although $\lceil N_I/K_I \rceil$ maximum flows and $K_I N_I$ comparisons in the process of looking for one maximum flow are necessary to finish the subchannel allocation, it needs to have $O(K_I N_I (\lceil N_I/K_I \rceil))$ operations to get the optional allocation solution.

However, there is a special situation when the maximum weighted edges of more than one SU connects to the same subchannel; in this case, K_I subchannels cannot be allocated to SUs when the maximum flow is obtained, the subchannel is only allocated to the SUs with the maximum weighted edge for all edges connected to the same subchannel. This will lead to the results:

(1) To complete the allocation, more than $\lceil N_I/K_I \rceil$ loops are necessary to acquire all maximum flows. In the worst case, N_I loops are necessary to get all maximum flows, so it needs to have $O(K_I N_I^2)$ operations to get the optional allocation solution. However, in this case, the computational complexity is just the same as the subchannel allocation algorithm proposed in [17].

(2) The subchannels allocated to SUs with good channel states may be far more than to SUs with poor channel states, which will lead to unfairness in resource allocation. To solve this problem, the poverty line constraint is adopted to improve fairness. The number of subchannels allocated to the k^{th} ($k=1,2,\dots,K_I$) SU is compared with the poverty line before allocating them, the SUs with enough subchannels will be removed from V_I and a new CRNs graph will be formed to find the optimal allocation. Therefore, it needs to have $O(K_I (N_I+1) (\lceil N_I/K_I \rceil))$ and $O(K_I N_I (N_I+1))$ operations to get the optional allocation solution in the best and worst cases, respectively.

The fairness function is adopted as the performance metric in subchannel allocation, the fairness function of different SUs is expressed as [21]

$$f(x) = \frac{\left(\sum_{i=1}^k R_i\right)^2}{\left(k \sum_{i=1}^k R_i^2\right)} \quad (8)$$

where k is the number of homogeneous SUs. Calculating the fairness function for different subchannel allocation algorithms, we think that the greater the $f(x)$ is, the better the fairness is.

3.2 Optimal Power Allocation

The optimal resource allocation problem at the sBS is equivalent to the optimal power allocation problem [22] when the optimal subchannel allocation is obtained, so we can rewrite the optimization problem (6) as

$$\begin{aligned} \max_{P_{k,n}} & \left\{ \sum_{k=1}^{K_1} \sum_{n \in \Omega_k} W \log_2 \left(1 + \frac{1.5P_{k,n} h_{k,n}^2 / N_0 W}{\ln(0.2/BER^{TAR})} \right) + \sum_{k=1}^{K_2} \sum_{n \in \Omega_k} W \log_2 \left(1 + \frac{1.5P_{k,n} h_{k,n}^2 / N_0 W + I_n}{\ln(0.2/BER^{TAR})} \right) \right\} \\ \text{s.t.} & \sum_{k=1}^K \sum_{n \in \Omega_k} P_{k,n} \leq P_{Total} \\ & P_{k,n} \geq 0 \forall k, \forall n, \\ & \sum_{k \in K_2} \sum_{n=1}^{N_2^m} \rho_{k,n} \sigma_{m,n}^2 P_{k,n} \leq \delta_m, m \in \mathbf{M} \\ & R_k \geq R_k^{\min} \forall k \in \mathbf{K}_1 \\ & R_1 : R_2 : \dots : R_{K_2} = r_1 : r_2 : \dots : r_{K_2} \forall k \in \mathbf{K}_2 \end{aligned} \quad (9)$$

where Ω_k is the set of subchannels assigned to the k^{th} SU. The minimum rate constraints given in (9) are nonconvex, and thus makes the optimization problem a nonconvex problem. However, it can be verified that the optimization problem satisfies the time-sharing condition [16] when the subcarriers go to infinity. The time-sharing condition implies that the maximum throughput is a concave function of $R = [R_1^{\min}, R_2^{\min}, \dots, R_K^{\min}]$, and the duality gap between the primal problem and its dual problem will be nearly zero. Therefore, we can solve the original optimization problem by considering its Lagrangian dual problem. The Lagrangian function of optimization problem (9) is described as:

$$\begin{aligned}
L(p_{k,n}, \lambda_1, \gamma_m, \beta_k, \mu_k) = & - \left\{ \sum_{k=1}^{K_1} \sum_{n \in \Omega_k} W \log_2 (1 + \Gamma_{k,n} P_{k,n}) \right. \\
& + \left. \sum_{k=1}^{K_2} \sum_{n \in \Omega_k} W \log_2 (1 + \Gamma_{k,n} P_{k,n}) \right\} \\
& + \lambda_1 \left(\sum_{k=1}^K \sum_{n \in \Omega_k} P_{k,n} - P_{Total} \right) + \sum_{m=1}^M \gamma_m \left(\sum_{k \in K_2} \sum_{n \in \Omega_k \in \lambda_2^m} \sigma_{m,n}^2 P_{k,n} - \delta_m \right) \\
& + \sum_{k=1}^{K_1} \beta_k \left(R_k^{\min} - \sum_{n \in \Omega_k} W \log_2 (1 + \Gamma_{k,n} P_{k,n}) \right) \\
& + \sum_{k=2}^{K_2} \mu_k \left\{ \sum_{n \in \Omega_1} W \log_2 (1 + \Gamma_{1,n} P_{1,n}) \right. \\
& \left. - \frac{r_1}{r_k} \sum_{n \in \Omega_k} W \log_2 (1 + \Gamma_{k,n} P_{k,n}) \right\}
\end{aligned} \tag{10}$$

where $\lambda_1, \gamma_m, \beta_k, \mu_k$ is the Lagrangian multiplier factors, and $\Gamma_{k,n} = \frac{1.5 h_{k,n}^2 / (N_0 W)}{\ln(0.2 / BER^{TAR})}, \forall k \in \mathbf{K}_1$

$$\Gamma_{k,n} = \frac{1.5 h_{k,n}^2 / (N_0 W + I_n)}{\ln(0.2 / BER^{TAR})}, \forall k \in \mathbf{K}_2.$$

Then, the Lagrangian dual function is expressed as

$$g(\lambda_1, \gamma_m, \beta_k, \mu_k) = \min_{p_{k,n}} L(p_{k,n}, \lambda_1, \gamma_m, \beta_k, \mu_k) \tag{11}$$

Then, the original optimization problem (9) can be expressed as a dual optimization problem as

$$\begin{aligned}
& \max g(\lambda_1, \gamma_m, \beta_k, \mu_k) \\
& s.t. \quad \lambda_1 \geq 0, \gamma_m \geq 0, \beta_k \geq 0
\end{aligned} \tag{12}$$

A dual decomposition method introduced in [23] is adopted to solve problem (12).

It is observed that the dual optimization problem (12) can be rewritten as

$$g(\lambda_1, \gamma_m, \beta_k, \mu_k) = \sum_{k \in \mathbf{K}_1} g_k^{(1)}(\lambda_1, \beta_k) + \sum_{k \in \mathbf{K}_2} g_k^{(2)}(\lambda_1, \gamma_m, \mu_k) - \lambda_1 P_{total} - \sum_{m=1}^M \gamma_m \delta_m \tag{13}$$

where

$$\begin{aligned}
& g_k^{(1)}(\lambda_1, \beta_k) = \\
& \min_{P_{k,n}} \left\{ - \sum_{n \in \Omega_k} W \log_2 (1 + \Gamma_{k,n} P_{k,n}) + \lambda_1 \sum_{n \in \Omega_k} P_{k,n} \right. \\
& \left. + \beta_k \left(R_k^{\min} - \sum_{n \in \Omega_k} W \log_2 (1 + \Gamma_{k,n} P_{k,n}) \right) \right\}
\end{aligned} \tag{14}$$

$$\begin{aligned}
& g_k^{(2)}(\lambda_1, \gamma_m, \mu_k) = \\
& \min_{P_{k,n}} \left\{ - \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) + \sum_{m=1}^M \sum_{n \in \Omega_k \in N_2^m} \gamma_m P_{k,n} \sigma_{m,n}^2 \right. \\
& \left. + \lambda_1 \sum_{n \in \Omega_k} P_{k,n} + \mu_k \left(\sum_{n \in \Omega_1} W \log_2(1 + \Gamma_{1,n} P_{1,n}) - \frac{r_1}{r_k} \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \right) \right\} \quad (15)
\end{aligned}$$

where $P_{k,n} \geq 0$, for $\forall k, \forall n$.

Therefore, for a given λ_1 and γ_m , problem (13) can be decomposed into K independent sub-problems as follows:

Sub-Problem 1 (SP1):

$$\begin{aligned}
& \min_{P_{k,n}} \left(- \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) + \lambda_1 \sum_{n \in \Omega_k} P_{k,n} \right) \quad \forall k \in K_1 \\
& \text{s.t.} \quad \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \geq R_k^{\min} \quad (16)
\end{aligned}$$

Sub-Problem 2 (SP2):

$$\begin{aligned}
& \min_{P_{k,n}} \left(- \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) + \lambda_1 \sum_{n \in \Omega_k} P_{k,n} + \sum_{m=1}^M \sum_{n \in \Omega_k \in N_2^m} \gamma_m P_{k,n} \sigma_{m,n}^2 \right) \quad \forall k \in K_2 \\
& \text{s.t.} \quad \sum_{n \in \Omega_1} W \log_2(1 + \Gamma_{1,n} P_{1,n}) = \frac{r_1}{r_k} \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \quad (17)
\end{aligned}$$

where SP1 and SP2 contain K_1 and K_2 independent optimization problems, respectively.

The Lagrangian problem for SP1 is

$$L(p_{k,n}, \beta_k) = - \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) + \lambda_1 \sum_{n \in \Omega_k} P_{k,n} + \beta_k \left(R_k^{\min} - \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \right) \quad (18)$$

Thus, the Karush–Kuhn–Tucher (KKT) conditions [16] of SP1 can be written as

$$\beta_k \left(R_k^{\min} - \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \right) = 0 \quad (19)$$

$$\frac{\partial L(p_{k,n}, \beta_k)}{\partial p_{k,n}} = 0, \quad \forall k \in K_1 \quad (20)$$

From the KKT conditions listed above, we can obtain the optimal power allocation $p_{k,n}^*$ for SP1 as:

$$p_{k,n}^* = \max\{p_{k,n}, 0\} \quad (21)$$

where $p_{k,n}$ is the following equation

$$p_{k,n} = \frac{W(1 + \beta_k)}{\lambda_1 \ln 2} - \frac{1}{\Gamma_{k,n}} \quad (22)$$

The Lagrangian problem for SP2 is

$$\begin{aligned}
L(p_{k,n}, \mu_k) = & \\
& - \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) + \lambda_1 \sum_{n \in \Omega_k} P_{k,n} + \sum_{m=1}^M \sum_{n \in \Omega_k \in \mathbb{N}_2^m} \gamma_m P_{k,n} o_{m,n}^2 \\
& + \mu_k \left(\sum_{n \in \Omega_1} W \log_2(1 + \Gamma_{1,n} P_{1,n}) - \frac{r_1}{r_k} \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \right)
\end{aligned} \tag{24}$$

Thus, the KKT conditions of SP2 can be written as

$$\mu_k \left(\sum_{n \in \Omega_1} W \log_2(1 + \Gamma_{1,n} P_{1,n}) - \frac{r_1}{r_k} \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \right) = 0 \tag{25}$$

$$\frac{\partial L(p_{k,n}, \mu_k)}{\partial p_{k,n}} = 0, \quad \forall k \in \mathbf{K}_2 \tag{26}$$

From the KKT conditions listed above, we can obtain the optimal power allocation $p_{k,n}^*$ for SP2 as:

$$p_{k,n}^* = \max\{p_{k,n}, 0\} \tag{27}$$

where $p_{k,n}$ is the following equation

$$P_{1,n} = \frac{W \left(1 - \sum_{k=2}^{K_2} \mu_k \right)}{\left(\lambda_1 + \gamma_m o_{m,n}^2 \right) \ln 2} - \frac{1}{\Gamma_{1,n}} \tag{28}$$

$$P_{k,n} = \frac{W (1 + \mu_k)}{\left(\lambda_1 + \gamma_m o_{m,n}^2 \right) \ln 2} - \frac{1}{\Gamma_{k,n}} \quad k \neq 1 \tag{29}$$

where $P_{k,n}$ is the function of Lagrangian multiplier factors achieved by subgradient algorithm. The whole procedure of the optimal power allocation algorithm is summarized as [Table 2](#).

Table 2. Power allocation algorithm

1: Initialization $\lambda_{i,1}, \varepsilon, j=1, a$
2: Repeat
3: Initialization $\beta_{k,i} \forall k \in \mathbf{K}_1, i=1$
4: $\forall k \in \mathbf{K}_1$, repeat
5: Calculate $p_{k,n} \forall n \in \Omega_k$, update $\beta_{k,i+1}$ by $\beta_{k,i+1} = \beta_{k,i} + a \left(R_k^{\min} - \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \right)$
6: if $\beta_{k,i+1} \leq 0$, set $\beta_{k,i+1} = 0$ and stop Otherwise, stop when $ \beta_{k,i+1} - \beta_{k,i} \leq \varepsilon$
7: Initialization $\gamma_{m,1}, \forall m \in M, \tau=1$
8: Repeat
9: Initialization $\mu_{k,1} \forall k \in \mathbf{K}_2, \sigma=1$
10: $\forall k \in \mathbf{K}_2$, repeat
11: Calculate $p_{k,n} \forall n \in \Omega_k$, update $\mu_{k,\sigma+1}$ by $\mu_{k,\sigma+1} = \mu_{k,\sigma} + a \left(\sum_{n \in \Omega_1} W \log_2(1 + \Gamma_{1,n} P_{1,n}) - \frac{r_1}{r_k} \sum_{n \in \Omega_k} W \log_2(1 + \Gamma_{k,n} P_{k,n}) \right)$
12: stop when $ \mu_{k,\sigma+1} - \mu_{k,\sigma} \leq \varepsilon$
13: Update $\gamma_{m,\tau+1}$ by $\gamma_{m,\tau+1} = \gamma_{m,\tau} + a \left(\sum_{k \in \mathbf{K}_2} \sum_{n \in \Omega_k \in \mathbb{N}_2^m} o_{m,n}^2 P_{k,n} - \delta_m \right)$
14: if $\gamma_{m,\tau+1} \leq 0$, set $\gamma_{m,\tau+1} = 0$ and stop

-
- Otherwise, stop when $|\gamma_{m \tau+1} - \gamma_{m \tau}| \leq \varepsilon$
- 15: Update $\lambda_{i, j+1}$ by
- $$\lambda_{i, j+1} = \lambda_{i, j} + a \left(\sum_{k=1}^K \sum_{n \in \Omega_k} P_{k,n} - P_{Total} \right)$$
-
- 16: if $\lambda_{i, j+1} \leq 0$, set $\lambda_{i, j+1} = 0$ and stop
- Otherwise, stop when $|\lambda_{i, j+1} - \lambda_{i, j}| \leq \varepsilon$
-
- where a is the step size, and $\varepsilon > 0$ is a given small constant.

3.3 HySOU Diversity Gain

The HySOU diversity gain is defined as the ratio of the total system throughput with the HySOU and the single sharing transmission mode, respectively, which is described as follows.

$$G = \frac{\sum_{k=1}^{K_1} \sum_{n \in \Omega_k \in N_1} W \log_2 (1 + \Gamma_{k,n} P_{k,n}) + \sum_{k=1}^{K_2} \sum_{n \in \Omega_k \in N_2} W \log_2 (1 + \Gamma_{k,n} P_{k,n})}{\sum_{k=1}^K \sum_{n \in \Omega_k \in \Lambda} W \log_2 (1 + \Gamma_{k,n} P_{k,n})} \quad (30)$$

where N_1 and N_2 are the number of idle and busy subchannels, respectively, and $\Lambda = N_1$ or N_2 , $P_{k,n}$ is the optimal transmission power of the k^{th} SU on the n^{th} subchannel.

4. Simulation Results and Analysis

In this section, some simulation results are presented to verify the performance of the proposed resource allocation algorithm. The subchannel from the sBS to every SU and PU, and from PU to every SU, is a Rayleigh fading channel and independent identically distributed (i.i.d). Four SUs, of which two are RT SUs and two are NRT SUs, are randomly located in the communication area, and communicate with the sBS through the subchannels licensed to the PN in which $M = 2$ PUs is assumed. We assume that the total number of subchannels is $N = 16$, of which four are idle subchannels and twelve are busy subchannels. The subchannel bandwidth is 20kHz, the noise power spectrum density on each subchannel is -100dBm .

In **Fig. 3**, we present the performance comparison of the HySOU algorithm and the overlay sharing mode algorithm. $R_k^{\min} = [R_1^{\min}, R_2^{\min}]$ are the minimum rate constraints of the K_1 RT SUs, set as $R_k^{\min} = [5, 6]$. As shown in the figure, with the increase of total power, the total throughput of the HySOU algorithm is significantly better than the overlay sharing mode algorithm. The total throughput of the overlay sharing mode algorithm is lower than 11 bit/s/Hz when the total power is less than 4W, which means that the CRNs cannot provide a satisfactory service for the RT SUs when the total power is low. However, the total throughput of the HySOU algorithm is more than 11 bit/s/Hz, even though the total power is less than 4W. It proves that the HySOU algorithm can obtain a better performance under the constraints.

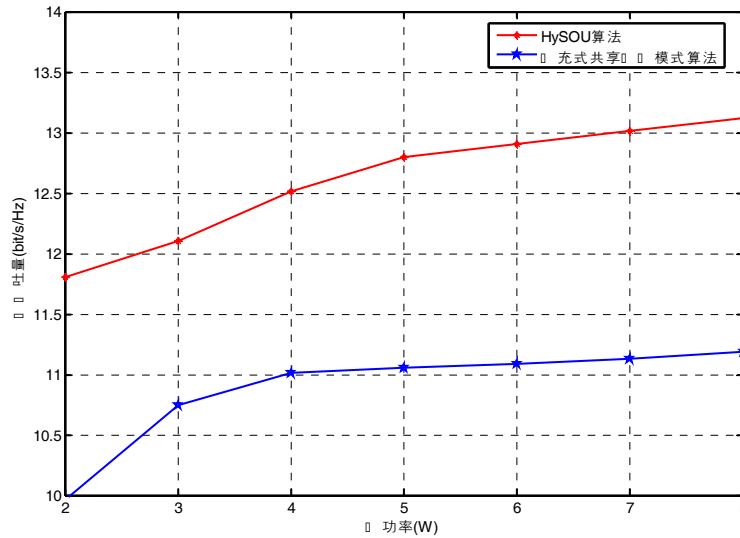
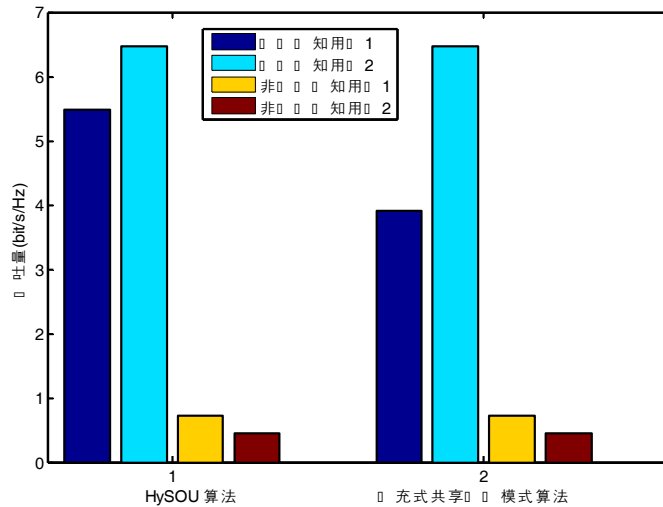
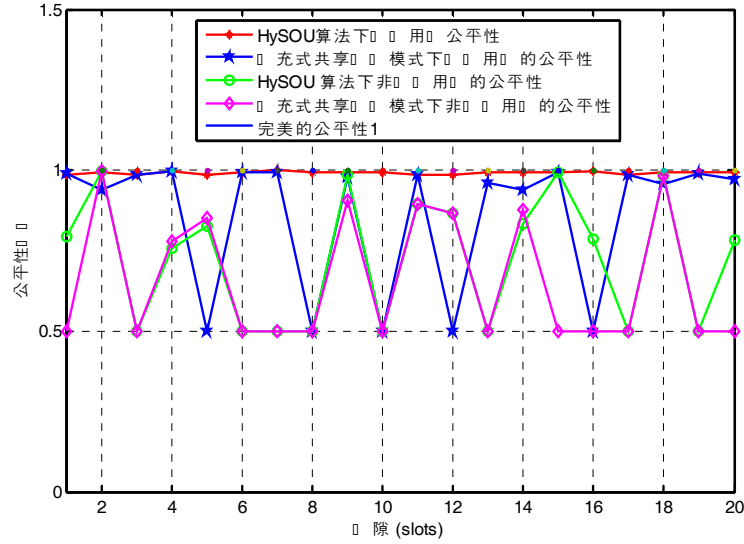


Fig. 3. Throughput comparison of HySOU algorithm and overlay sharing mode algorithm

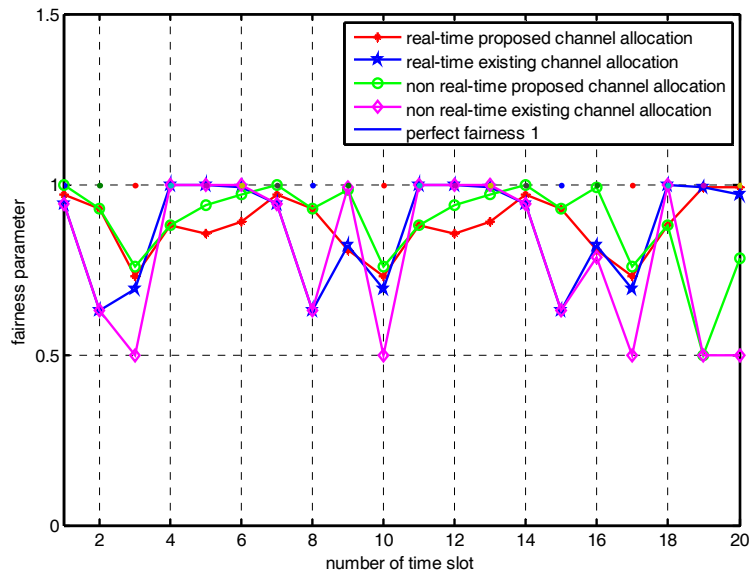
Fig. 4 shows the fairness comparison of the HySOU algorithm and overlay sharing mode algorithm. The throughput of every SU is shown in Fig. 4 (a); it shows that the throughput difference between the homogeneous SUs with the HySOU algorithm is smaller than the overlay sharing mode algorithm. The fairness function of RT and NRT SUs are given in Fig. 4 (b) and Fig. 4 (c). In Fig. 4 (b), the minimum rate constraints of the K_1 RT SUs are set as $R_k^{\min}=[5,6]$, and in Fig. 4 (c), the minimum rate constraints of the K_1 RT SUs are set as $R_k^{\min}=[5,20]$. In Fig. 4 (b), with the increase of the number of time slots, the fairness function of homogeneous SUs with the HySOU algorithm is always better than the overlay sharing mode algorithm, especially for the RT SUs with the HySOU algorithm, where the fairness function converges at perfect fairness. In Fig. 4 (c), as the difference between the required minimum rates of RT SUs become larger, similarly, the fairness function of homogeneous SUs with the HySOU algorithm is better than the overlay sharing mode algorithm with the increase in the number of time slots.



(a)

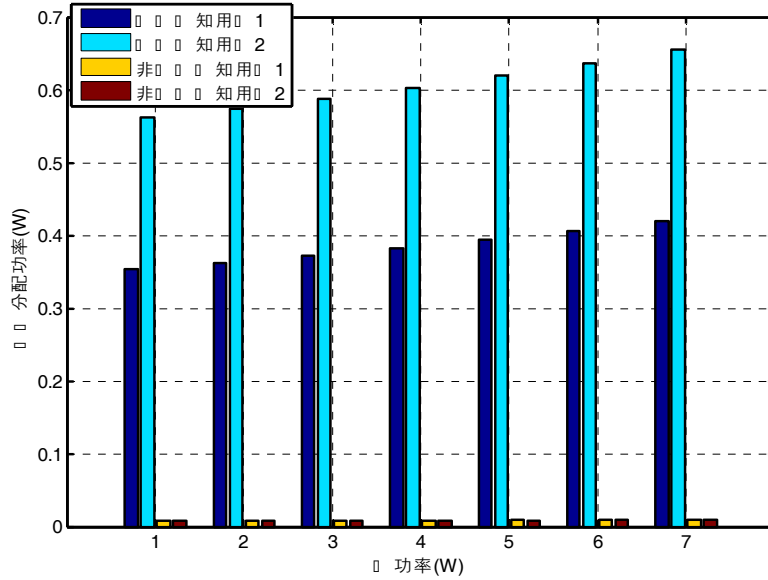


(b)

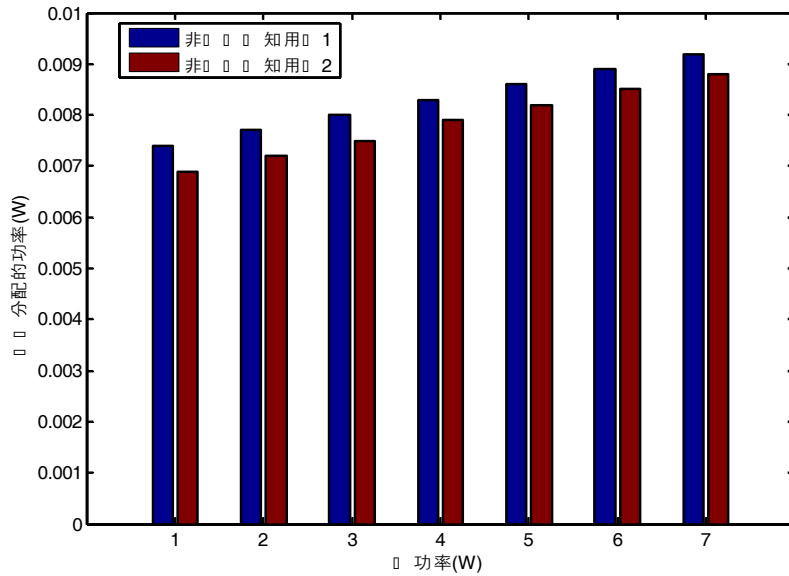


(c)

Fig. 4. Fairness comparison of the HySOU algorithm and overlay sharing mode algorithm. (a) Throughput of every SU; (b) Fairness function of RT and NRT SU; (c) Fairness function of RT and NRT SU



(a)



(b)

Fig. 5. The power allocated to SUs. (a) The power allocated to the RT and NRT SUs; (b) Amplificatory histogram of the power allocated to NRT SUs

The power allocated to every SU is shown in **Fig. 5**. As shown in the figure, with total power increasing, the power allocated to every SU increases. The increase amplitude of the power allocated to RT SUs is bigger than the NRT SUs, because the power of the NRT SUs is strictly limited below the interference thresholds of the PUs.

Fig. 6 illustrates the total throughput comparison with different minimum rate constraints for RT SUs. The minimum rate constraints of the RT SUs are set as $R_k^{\min}=[5,6]$, $R_k^{\min}=[7,8]$

and $R_k^{\min}=[9,10]$, respectively. As shown in the figure, with the maximal minimum rate constraints, the total throughput is below the minimum rate requirements when the total power is lower than 5W. For different minimum rate constraints, the minimum rate requirements are just satisfied when the total power reach a power range, and exceeds the power range when the increased amplitude of total throughput changes slowly with the increase of total power.

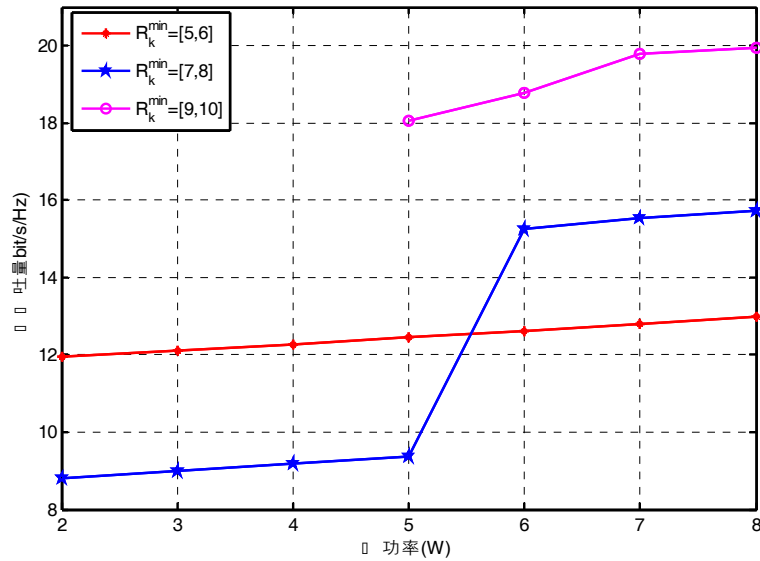


Fig. 6. Total throughput comparison with the minimum rate constraint.

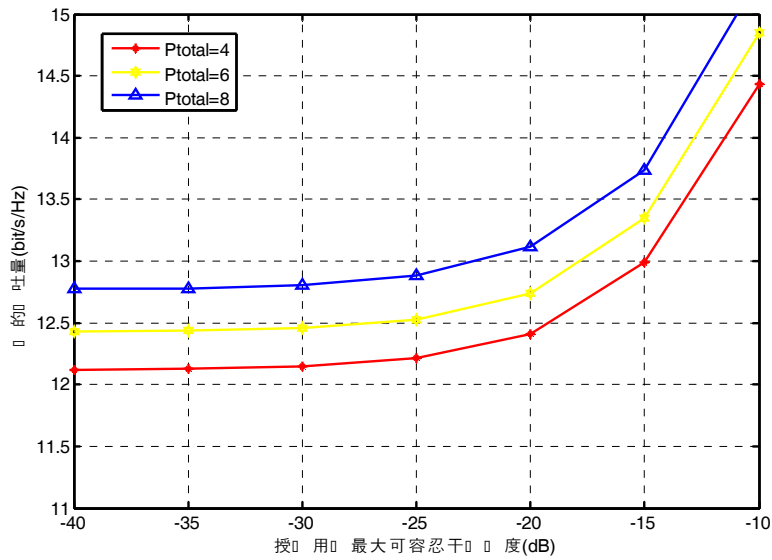


Fig. 7. Total throughput with maximal interference temperature of PU.

The total throughput performance with different maximal interference temperatures of the PU is shown in Fig. 7. The total throughput significantly increases after the maximal interference temperature of the PU reaches -20dB. However, the increase of the total

throughput is as slow as the increase of maximal interference temperature of the PU before this value.

The HySOU diversity gain is shown in Fig. 8, in which the blue and red lines are the HySOU diversity gain when R_{single} is the system throughput with the underlay sharing mode and the overlay sharing mode, respectively. From the figure, we find that the gain increases as the total power increases, because the power allocated to the SUs is limited by the interference temperature of PUs with underlay sharing mode, so the system throughput with underlay sharing mode is lower than the overlay sharing mode, which leads to the blue line being higher than the red line.

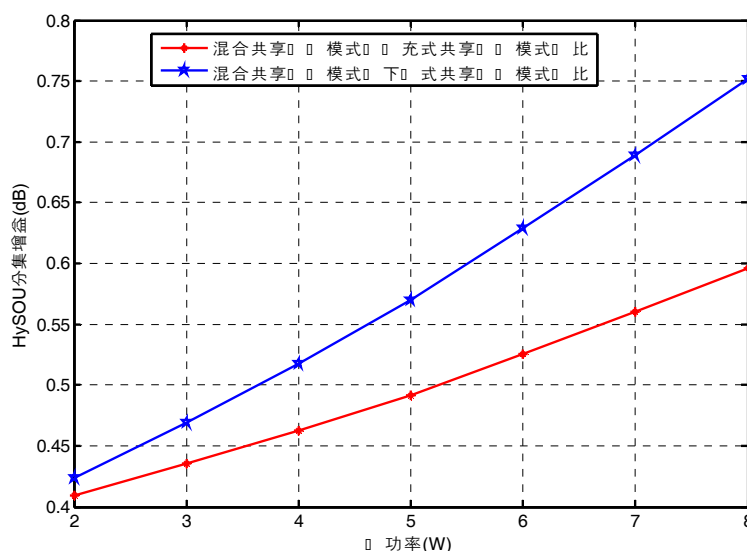


Fig. 8. HySOU diversity gain with different sharing modes.

5. Conclusion

In this paper, we have studied the resource allocation problem in OFDM CRNs that support heterogeneous services. To utilize the radio resources fully and improve the spectrum utilization, we assign different subchannels (the idle and busy subchannels) to different SUs, according to their QoS requirements, and we have formulated the problem of resource allocation as a mixed-integer programming problem. For simplicity, this problem is divided into two sub-problems: the subchannel allocation and the power allocation. We have proposed a novel resource allocation algorithm called the HySOU algorithm to solve this problem. Finally, the simulation results are illustrated to demonstrate the performance of the proposed HySOU algorithm. Based on the simulation results, we have verified that the HySOU algorithm has a better performance in improving the spectrum utilization.

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