Joint Relay-and-Antenna Selection and Power Allocation for AF MIMO Two-way Relay Networks

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Abstract

In this paper, we present a joint relay-and-antenna selection and power allocation strategy for multiple-input multi-output (MIMO) amplify-and-forward (AF) two-way relay networks (TWRNs). In our approach, we select the best transmit and receive antennas at the two sources, a best relay and a best transmit and receive antenna at the selected relay based on maximizing the minimum of the end-to-end received signal-to-noise-ratios (SNRs) under a total transmit power constraints. We obtained the closed-form solution for the optimal power allocation firstly. Then with the optimal allocation solution we found, we can reduce the joint relay-and-antenna selection to a simpler problem. Besides, the overall outage probability is investigated and a tight closed-form approximation is derived, which provides a method to evaluate the outage performance easily and fast. Simulation results are presented to verify the analysis.

Keywords: Joint relay-and-antenna selection, MIMO, power allocation, outage performance, two-way relay networks, amplify-and-forward
1. Introduction

Cooperative relaying has been shown to be an effective means to extend service coverage and increase system capacity [1-5]. In order to improve the drawbacks of low spectral efficiency for the traditional one-way relaying, several techniques have been proposed, such as full-duplex (FD) technique [6-8] and two-way relaying [9]. Especially, two-way relaying is a promising spectral efficient transmission protocol for wireless networks with half-duplex terminals. By using network coding [10] at the two-way relay, where multiple data streams arriving from multiple sources are mixed or network-coded before being broadcasted, the transmission phases required is reduced, resulting in an improved spectral efficiency. At the destination, upon receiving the broadcast signals from the relay, each destination can decode its intended information by using self-interference cancellation.

The performance of TWRNs can be improved by integrating multiple-input multiple-output (MIMO) transmission technology [11]. However, the main drawback of any MIMO system is the increased system complexity due to the additional cost for enabling transmit and receive radio frequency (RF) chains. Antenna selection (AS) [12] scheme has been proposed as a promising technique to achieve antenna diversity in MIMO system with reduced hardware complexity. Although it has inferior performance compared to beamforming-based schemes, it has the advantage of tractability and low implementation complexity.

The impact of antenna selection in multiple-antenna two-way relay networks is considered in [13-16]. Reference [13-14] investigate antenna selection for decode-and forward (DF) TWRNs. [13] proposed the use of Max-Min criteria for antenna selection with binary network coding and transmit beamforming without binary network coding. [14] analyzed the performance of a two-way relay system using Alamouti coding (open loop) and antenna selection (close loop) with network coded messages. Then antenna selection for AF MIMO TWRNs is studied in [15-16]. [15] developed a computationally efficient scheme to select a subset of relay antennas to maximize the achievable sum rate (ASR) of AF MIMO two-way relay channels. Then in [16], a new framework for the comparative analysis of beamforming and antenna selection with non-identical fading parameter $m$ in the two source-relay links is presented. However, the system models in the papers above employ multiple antennas either at the relay or at the sources only. Furthermore, the antenna selection is considered at the sources or the relay only.

For the sake of completeness, in [17-19], end-to-end antenna selection for AF MIMO TWRNs is studied. Two new transmit/receive (Tx/Rx) antenna selection strategies are proposed and analyzed for two-way MIMO AF relay networks in [17-18]. These two strategies select the best transmit and receive antennas at the two sources and the relay based on (i) minimizing the overall outage probability and (ii) maximizing the sum-rate. The performance of these selection strategies is quantified by deriving the overall outage probability, its high SNR approximation and the diversity order. Then in [19], the impact of relay location and power allocation on the overall outage probability are studied. But it just optimized the power allocation approximately based on the expressions of the outage probability, and didn’t get the closed-form solution for the optimal power allocation solution.

From the analysis above, it can be concluded that joint end-to-end (E2E) antenna selections and power allocation strategy is rarely studied. We herein present a joint relay-and-antenna
selection and power allocation scheme which has a closed-form solution. With the optimal power allocation solution found, the joint relay-and-antenna selection problem can be reduced to a simpler problem, which makes this protocol easier and less expensive for implementation in distributed relay networks. Also, a tight closed-form approximation of the overall outage probability is derived under the optimal power allocation. Using the expressions, one can evaluate the outage performance of the proposed system easily and fast.

The rest of this paper is organized as follows. In section 2, we introduce the system model. In section 3, the closed-form solution of optimal power allocation strategy is analyzed first, then a simpler joint relay-and-antenna scheme is proposed. The outage performance analysis are carried out in section 4. Simulations results are presented in section 5 and conclusion are made in section 6.

2. System Model

We consider a MIMO AF TWRNs consisting of two transceivers (\(S_1\) and \(S_2\)) and \(K\) relays (\(R_k, k = 1, \ldots, K\)) as shown in Fig. 1. Specifically, \(S_1\), \(S_2\) and \(R_k\) are equipped with \(N_1\), \(N_2\) and \(N_{R_k}\) antennas, respectively. We assume the direct communication links between the two transceivers is not available because of the poor quality of the channel between them. Besides, all nodes are assumed to be half-duplex and all channel amplitude are assumed to be independently identically distributed frequency-flat Rayleigh fading. The feedbacks for antenna selection are assumed to be perfect unless otherwise stated. The channel matrix from \(S_1\) to \(R_k\) is denoted by \(H_{S,R_k}\). All the channels coefficients are assumed to be fixed over two consecutive time slots unless otherwise stated. Thus the channel matrix from \(R_k\) to \(S_1\) can be denoted as \(\left(H_{S,R_k}\right)^T\). Furthermore, the \((n_k, l)\)-th element of \(H_{S,R_k}\) is denoted by \(h_{S,R_k}^{n_k,l}\) and modeled as \(h_{S,R_k}^{n_k,l} \sim \text{CN}(0,1)\).

![Fig. 1. System Model](image-url)
In this protocol, $S_1$ and $S_2$ exchange their information $x_1$ and $x_2$ in two consecutive time-slots. Applying the proposed joint relay-and-antenna selection scheme, a single antenna at the two source nodes, a best relay and a best antenna at the selected relay are selected. Without loss of generality, we assume the $j$-th and $l$-th transmit antennas at $S_1$ and $S_2$, relay $R_k$, the $n_{R_k}$-th antenna of $R_k$ are selected. In the first time-slot, both $S_1$ and $S_2$ transmit $x_1$ and $x_2$ simultaneously to the relays using the selected antennas. $R_k$ receives the superimposed-signal using the $n_{R_k}$-th antenna, then the received signal can be expressed as follows:

$$y_{R_k} = \sqrt{P_1} h_{S_1 R_k}^{n_{R_k}} j x_1 + \sqrt{P_2} h_{S_2 R_k}^{n_{R_k}} j x_2 + n_{R_k}$$

(1)

where $P_1$ and $P_2$ are the transmit powers of transceivers $S_1$ and $S_2$, respectively, and $n_{R_k}$ is the additive white Gaussian noise (AGWN) at $R_k$ with zero-mean and unit variance.

In the second time-slot, relay $R_k$ amplifies $y_{R_k}$ with a complex weight $\omega_k$ and then broadcast it again using the $n_{R_k}$-th antenna to $S_i \left[ i = 1, 2 \right]$ over a broadcast channel. Then, $S_1$ and $S_2$ receive the signal again using the $j$-th and $l$-th receive antennas respectively, thus, the signal received at $S_1$ and $S_2$ can be represented respectively as

$$y_{S_1} = \sqrt{P_1} \omega_k \left[ h_{S_1 R_k}^{n_{R_k}} j \right]^2 x_1 + \sqrt{P_2} \omega_k h_{S_1 R_k}^{n_{R_k}} j h_{S_2 R_k}^{n_{R_k}} j x_2 + \omega_k h_{S_1 R_k}^{n_{R_k}} j n_{R_k} + n_{S_1}$$

(2)

$$y_{S_2} = \sqrt{P_1} \omega_k h_{S_1 R_k}^{n_{R_k}} j h_{S_2 R_k}^{n_{R_k}} j x_1 + \sqrt{P_2} \omega_k \left[ h_{S_2 R_k}^{n_{R_k}} j \right]^2 x_2 + \omega_k h_{S_2 R_k}^{n_{R_k}} j n_{R_k} + n_{S_2}$$

(3)

where $n_{S_i} \mid i = 1, 2$ is the noise at the two transceiver respectively, and they are assumed to be i.i.d. Gaussian with zero-mean and unit variance. The first term in (2), known as self-interference, can be subtracted from $y_{S_1}$. Similarly, the second term in (3) can be subtracted from $y_{S_2}$. The residual signals $\tilde{y}_{S_1}$ and $\tilde{y}_{S_2}$ after self-interference cancellation are expressed as

$$\tilde{y}_{S_1} = y_{S_1} - \sqrt{P_1} \omega_k \left[ h_{S_1 R_k}^{n_{R_k}} j \right]^2 x_1 = \sqrt{P_2} \omega_k h_{S_1 R_k}^{n_{R_k}} j h_{S_2 R_k}^{n_{R_k}} j x_2 + \omega_k h_{S_1 R_k}^{n_{R_k}} j n_{R_k} + n_{S_1}$$

(4)

$$\tilde{y}_{S_2} = y_{S_2} - \sqrt{P_2} \omega_k \left[ h_{S_2 R_k}^{n_{R_k}} j \right]^2 x_2 = \sqrt{P_1} \omega_k h_{S_1 R_k}^{n_{R_k}} j h_{S_2 R_k}^{n_{R_k}} j x_1 + \omega_k h_{S_2 R_k}^{n_{R_k}} j n_{R_k} + n_{S_2}$$

(5)

The residual signals can be used to decode the information symbols at $S_1$ and $S_2$ respectively.

Then the E2E SNR at $S_1$ and $S_2$ can be derived as follows

In this section, a joint relay-and-antenna selection and power allocation strategy is proposed for MIMO AF TWRNs. The key design criterion is the joint selection of the best relay, single transmit and receive antenna at $S_1, S_2$ and the selected relay, and optimal power allocation to maximize the smaller of the E2E SNR, i.e., minimizing the overall outage probability.

The network is assumed to have a total transmit power constraint $P_T$. Denoting the transmit power at $R_k$ is $P_{R_k}$ and assuming that the information symbols and noises are independent, using (1), it can be shown that

$$
SNR_{S_1}^{j,l,n_k} = \frac{P_1 |\omega_k|^2 |h_{S_1,R_k}^{n_k,j} h_{S_1,J}^{n_k,l}|^2}{1 + |\omega_k|^2 |h_{S_1,J}^{n_k,l}|^2} \tag{6}
$$

$$
SNR_{S_2}^{j,l,n_k} = \frac{P_2 |\omega_k|^2 |h_{S_2,J}^{n_k,l} h_{S_2,R_k}^{n_k,j}|^2}{1 + |\omega_k|^2 |h_{S_2,J}^{n_k,l}|^2} \tag{7}
$$

Then when the relay and antenna are selected as the assumption, the total transmit power can be written as

$$
P_{total} = P_1 \left(1 + |h_{S_1,J}^{n_k,l}|^2 |\omega_k|^2 \right) + P_2 \left(1 + |h_{S_2,J}^{n_k,l}|^2 |\omega_k|^2 \right) \tag{9}
$$

Then the joint relay-and-antenna selection and power allocation problem using Max-Min criteria under the total power constraint $P_T$ can be represented as

$$
\max_{P_1,P_2,\omega_k,j,l,k,n_k} \min(SNR_{S_1}^{j,l,n_k},SNR_{S_2}^{j,l,n_k}) \quad \text{subject to} \quad P_{total} \leq P_T \tag{10}
$$

Equation (10) is equivalent to the following expressions

$$
\max_{P_1,P_2,\omega_k,j,l,k,n_k} \quad \text{subject to} \quad SNR_{S_1}^{j,l,n_k} \geq \alpha, SNR_{S_2}^{j,l,n_k} \geq \alpha, P_{total} \leq P_T \tag{11}
$$

The joint optimization problem is equivalent to first optimizing over $P_1, P_2, \omega_k$, which is the optimal power allocation problem, then optimizing over $j,l,k,n_k$, which is the optimal relay-and-antenna selection problem. In the following, we consider the two problems separately.
3.1 Optimal power allocation strategy

Firstly we consider the optimal power allocation problem, and it can be represented as

$$\max_{P_1, P_2, \omega, J} t \quad \text{subject to} \quad SNR_{S_1}^{j, \omega} \geq t, \; SNR_{S_2}^{j, \omega} \geq t, \; P_{\text{total}} \leq P_T \quad (12)$$

It is a constrained optimization and can be solved using the SNR balancing technique [20].

As shown in [20], it is required that $SNR_{S_i}^{j, \omega} = SNR_{S_2}^{j, \omega}$. Using (6) and (7), we can obtain the following equation

$$P_2 \left(1 + |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2\right) = P_1 \left(1 + |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2\right) \quad (13)$$

Using (13), the optimization problem (12) can be written as follows

$$\max_{P_1, P_2, \omega, J} t \quad \text{subject to} \quad SNR_{S_1}^{j, \omega} = \frac{P_1 |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2}{1 + |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2} = t$$

$$P_{\text{total}} = 2P_1 \left(1 + \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2 |\omega_k|^2\right) + |\omega_k|^2 \leq P_T \quad (14)$$

Combining the two constraints in (14) leads to the following optimization problem

$$\max_{\omega, J} t \quad \text{subject to} \quad \frac{2t \left(1 + |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2\right) + |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2}{1 + |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2} \left(1 + |\frac{n_{S_k,R_k}}{h_{S_2,R_k}}|^2 |\omega_k|^2\right) + |\omega_k|^2 \leq P_T \quad (15)$$

The equation above is equivalent to the following expressions

$$\max_{\omega, J} \frac{\left(P_T - |\omega_k|^2\right)\left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2}{1 + |\frac{n_{S_k,R_k}}{h_{S_2,R_k}} |^2 |\omega_k|^2} \left(1 + |\omega_k|^2 \left|\frac{n_{S_k,R_k}}{h_{S_2,R_k}}\right|^2\right) \quad (16)$$

From the formula above, we can see that the objective function does not depend on the phase of $\omega_k$, therefore, no phase adjustment is required at the relay. Differentiating the objective function and equating it to zero lead us to the following equation:
Calculate the positive solution to (17), and we can get the optimal amplification coefficient at relay $R_k$ as

$$\omega_k^{\text{opt}} = \frac{P_T}{\sqrt{1 + P_T \left| h_{S_k,R_k}^{n_k,j} \right|^2 \left( 1 + P_T \left| h_{S_k,R_k}^{n_k,j} \right|^2 \right)}}$$

(18)

Using (9) and (13) along with the fact that at the optimum the power constraint in (12) holds with equality, we can get

$$P_T = 2P_1 \left( 1 + \left| h_{S_1,R_k}^{n_k,j} \right|^2 \left| \omega_k \right|^2 \right) + \left| \omega_k \right|^2 = 2P_2 \left( 1 + \left| h_{S_2,R_k}^{n_k,j} \right|^2 \left| \omega_k \right|^2 \right) + \left| \omega_k \right|^2$$

(19)

From the formula above, we can obtain the optimal power allocated to the two sources, respectively, as

$$P_1^{\text{opt}} = \frac{P_T \sqrt{1 + P_T \left| h_{S_1,R_k}^{n_k,j} \right|^2}}{2 \sqrt{1 + P_T \left| h_{S_1,R_k}^{n_k,j} \right|^2} + 2 \sqrt{1 + P_T \left| h_{S_2,R_k}^{n_k,j} \right|^2}}$$

$$P_2^{\text{opt}} = \frac{P_T \sqrt{1 + P_T \left| h_{S_2,R_k}^{n_k,j} \right|^2}}{2 \sqrt{1 + P_T \left| h_{S_1,R_k}^{n_k,j} \right|^2} + 2 \sqrt{1 + P_T \left| h_{S_2,R_k}^{n_k,j} \right|^2}}$$

(20)

(21)

where $P_1^{\text{opt}}$ and $P_2^{\text{opt}}$ denote the optimal value of $P_1$ and $P_2$, respectively. Interestingly, $P_1^{\text{opt}} + P_2^{\text{opt}} = 0.5P_T$ and thus $P_{R_k} = 0.5P_T$, which shows that with the optimal power allocation, the total transmit power is shared equally between the two transceivers on one side and the $k$-th relay on the other side.

### 3.2 Joint relay-and-antenna selection scheme with optimal power allocation

In this subsection, we proposed a joint relay-and-antenna selection scheme with low complexity under the optimal power allocation solutions obtained in section 3.1. Substituting (21) and (18) into (7), we can obtain the maximum balanced SNR, achieved by selecting the $j$-th and $l$-th transmit antennas at $S_1$ and $S_2$, relay $R_k$, the $n_k$-th antenna of $R_k$, as follows
In this case, the Max-Min criteria can be simplified to the following problem

\[
\mathbf{j}^*, \mathbf{l}^*, k^*, n^*_k = \arg \max_{j, l, k, n_k} SNR_{b_{j, l, n_k}}^{j, l, n_k}
\]  

Then we just need to calculate the balanced SNR to achieve the joint relay-and-antenna selections which results in low computational complexity. Besides, as the transmit power constraint \( P_T \) is known to the relays, and they can get the channel information from itself to the sources, distributed timer technique (DT) \[21\] can be adopted in the joint relay-and-antenna selection. And it can be conducted as follows:

Step 1: The two sources send pilot information to all the relays, and relay \( R_k \) \((k = 1, \ldots, K)\) can get the channel matrix between \( S_1 \) to \( R_k \), which are denoted by \( H_{S_1, R_k} \) and \( H_{S_2, R_k} \).

Step 2: As \( P_T \) is known to the relays, \( R_k \) can calculate the balanced SNR \( SNR_{b_{j, l, n_k}}^{j, l, n_k} \) when the \( j \)-th, \( l \)-th and \( n_k \)-th antenna is selected at \( S_1 \), \( S_2 \) and \( R_k \) respectively. Obviously, \( R_k \) need to computer \( N_1 \times N_2 \times N_{R_k} \) balanced SNRs. Then \( R_k \) start up the timer, and the timing is set up to \( 1/\max_{j, l, n_k} \left( \text{max } \text{SNR}_{b_{j, l, n_k}}^{j, l, n_k} \right) \), where \( \text{max } \text{SNR}_{b_{j, l, n_k}}^{j, l, n_k} \) denotes the maximum of the \( N_1 \times N_2 \times N_{R_k} \) balanced SNRs at \( R_k \), and the corresponding \( j, l, n_k \) denotes the optimal antenna index \( j^*, l^*, n^*_k \) if \( R_k \) is the optimal relay.

Step 3: The relay which has the shortest time will broadcast its index, and the other relays back off. Without loss of generality, we assume \( R_k \) expires first, it broadcast \( k \) and the other relays keep silent after receiving the broadcast information. Then \( R_k \) broadcast the optimal antenna index \( j^*, l^* \) by which the sources can select the best antenna.

When \( P_T >> 1/\left| h_{S_1, R_k}^{n_k} \right|^2 \) and \( P_T >> 1/\left| h_{S_2, R_k}^{n_k} \right|^2 \), we have

\[
SNR_{b_{j, l, n_k}}^{j, l, n_k} = SNR_{S_1}^{j, l, n_k} = SNR_{S_2}^{j, l, n_k} = \frac{P_T \left| h_{S_1, R_k}^{n_k} \right|^2}{2 \left( \left| h_{S_1, R_k}^{n_k} \right|^2 + \left| h_{S_2, R_k}^{n_k} \right|^2 \right)} = \frac{P_T}{2 \left( \frac{1}{\left| h_{S_1, R_k}^{n_k} \right|^2} + \frac{1}{\left| h_{S_2, R_k}^{n_k} \right|^2} \right)}
\]  

(24)
It is observed that when $P_T$ is large enough, the balanced SNR increase linearly with $P_T$. Besides, as can be observed from (24), when $P_T$ is large, we can jointly select the relay and antenna using the harmonic mean of the magnitude of the channel coefficients, i.e. $\left| \hat{h}_{S_R_k}^{n_j, k} \right|$ and $\left| \hat{h}_{S_{R_k}}^{n_l, k} \right|$. The best relay-and-antenna corresponds to the largest values of the harmonic mean. Certainly, DT technique also can be used in this case, and we just need to set the timing at relay $R_k$ to $\min_{j,l, n_k} \left( \frac{1}{\left| \hat{h}_{S_{R_k}}^{n_j, k} \right|} + \frac{1}{\left| \hat{h}_{S_{R_k}}^{n_l, k} \right|} \right)$ in step 2.

So with the balanced SNR, the joint selection of the relay-and-antenna become easy to achieve and has low computation complexity, which results in reduction of the complexity and energy consumption for the wireless device.

4. Outage Performance Analysis with Optimal Power Allocation

In this section, the outage performance of the proposed strategy is studied. In particular, we present an easy-to-compute approximated expression for the exact outage probability to reduce the computational cost.

The overall outage probability ($P_{out}$) is defined as the probability that the instantaneous E2E SNR of the weakest source falls below a present threshold $\gamma_{th}$, and it can be given by

$$P_{out} = \Pr \left[ Z = \max_{1 \leq j \leq N_s, 1 \leq l \leq N_s} \left( \min_{1 \leq k \leq K, 1 \leq n_k \leq N_k} \left( \text{SNR}_{S_1}^{j, n_k}, \text{SNR}_{S_2}^{j, n_k} \right) \right) \leq \gamma_{th} \right] \tag{25}$$

Under the optimal power allocation, we have $\text{SNR}_{S_1}^{j, n_k} = \text{SNR}_{S_2}^{j, n_k} = \text{SNR}_b^{j, n_k}$, then the formula above can be expressed as

$$P_{out} = \Pr \left[ Z = \max_{1 \leq j \leq N_s, 1 \leq l \leq N_s} \left( \text{SNR}_b^{j, n_k} \right) \leq \gamma_{th} \right] = \Pr \left[ \max_{1 \leq k \leq K, 1 \leq n_k \leq N_k} \left( \text{SNR}_b^{j, n_k} \right) \leq \gamma_{th} \right] \tag{26}$$

where $\text{SNR}_b^{j, n_k}$ is defined as follows:

$$\text{SNR}_b^{j, n_k} = \max_{1 \leq j \leq N_s, 1 \leq l \leq N_s} \left( \text{SNR}_{b}^{j, n_k} \right) = \frac{P_T^2 X_{n_k} Y_{n_k}}{2 \left( \sqrt{1 + P_T X_{n_k}} + \sqrt{1 + P_T Y_{n_k}} \right)^2} \tag{27}$$

where $X_{n_k} = \max_{1 \leq j \leq N_s} \left( \left| \hat{h}_{S_{R_k}}^{n_j, k} \right|^2 \right)$ and $Y_{n_k} = \max_{1 \leq l \leq N_s} \left( \left| \hat{h}_{S_{R_k}}^{n_l, k} \right|^2 \right)$. 
We assume that \( P_{rX_{n2}} >> 1 \) and \( P_{rY_{n2}} >> 1 \), then

\[
\text{SNR}_{b} = Z_{n2} \approx \frac{P_{rX_{n2}}^2 \cdot Y_{n2}}{2(\sqrt{P_{rX_{n2}}} + \sqrt{P_{rY_{n2}}})^2}
\]  

(28)

In order to get the overall outage probability, the CDF of \( Z_{n2} \) should be first derived. Thus the CDF of \( Z_{n2} \) is given by (see Appendix for the proof)

\[
F_{Z_{n2}}(z) = 1 - \sum_{p=0}^{N_1-1} \sum_{q=0}^{N_2-1} N_1 N_2 \binom{N_1 - 1}{p} \binom{N_2 - 1}{q} (-1)^{p+q} \times \left( e^{-2(1+z)/P_r} + e^{-2(1+z)/P_T} - e^{-2(1+z)/P_r} - e^{-2(1+z)/P_T} \right)
\]

\[
= \left( \sum_{k=1}^{K} F_{Z_{n2}}(z) \right)^{N_{n2}}
\]

(29)

Define \( Z_{n2} = \max_{1 \leq n2 \leq N_{n2}} (Z_{n2}) \), by identifying that \( Z_{n2} |_{n2=1} \) are statistically independent identically distributed (i.i.d) random variables, the CDF of \( Z_{n2} |_{n2=1} \) can be derived readily as \( F_{Z_{n2}}(z) = \prod_{k=1}^{K} F_{Z_{n2}}(z) \).

The outage probability can be derived readily by evaluating the CDF of \( Z \) at the threshold \( \gamma_{th} \) as: \( P_{out} = F_{Z}(\gamma_{th}) \).

5. Numerical and Simulation Results

In this section we present the numerical and Monte-Carlo simulations results for the proposed joint relay-and-antenna selection and power allocation strategy. The target transmission rate is set as \( R = 1 \text{bit} / s / \text{Hz} \).

In Fig. 2, the overall outage probability of the proposed scheme is compared with those of other traditional algorithms. Specially, we compare our proposed joint relay-and-antenna selection technique with two other schemes: scheme 1 where optimal relay-and-antenna selection (the relay-and-antenna that results in the highest value of the smaller of the two SNRs is chosen [11]) is used with equal power allocation and scheme 2 where random relay-and-antenna selection is used with equal power allocation. For scheme with equal power allocation, 1/3 of the total available power is allocated to each of the two transceivers and the relay. As can be seen from the figure, a significant performance gain can be achieved via optimal power allocation compared with scheme 1. It is also observed that as the total transmission power increase, the performance advantage of optimal power allocation decreases, for example, the proposed scheme get about 1dB gain in the middle SNR regions while about 0.5 dB gain in the high SNR region. This is because that the increment of
$P_T$ reduces the effects of the channel gain gap between the links from the two sources to the relays, and if $P_T$ is big enough, the optimal power allocation is approximated to the equal power allocation scheme. Besides, it is clear that the approximated closed-form expression of the overall outage probability provides a good approximation to the simulated results especially in the high SNR regions. Scheme 2 has the worst outage performance because it can’t get any diversity order and coding gains.

**Fig. 2.** Outage probability for different transmission schemes versus $P_T$

**Fig. 3.** Outage probability versus $P_T$ for different relay configuration
Fig. 3 depicts the overall outage probability of the proposed strategy for different relay numbers versus $P_T$. Simulations assumed that $N_1 = N_2 = N_{R_k} = 2$. It clearly illustrates the performance gains of joint relay-and-antenna selection of multi-relay TWRNs over that of their single-relay counterpart. For example, at $10^{-2}$ outage probability, three-relay TWRNs provide a gain of 9.5dB over that of its single-relay counterpart. It also can be seen that the approximated closed-form analytical result provides a good approximation to the simulated results at almost all SNRs which verifies the correctness of the theoretical approximation.

Fig. 4 depicts the overall outage probability for different antenna configurations with $K=4$. It can be observed that increasing the number of the antenna both at the sources and the relays could improve the outage performance. For example, with outage probability $10^{-2}$, it can get 13dB gains when all the nodes are deployed with 3 antennas.

![Graph](image)

**Fig. 4.** Outage probability versus $P_T$ for different antenna configuration with optimal power allocation.

Fig. 5 gives the maximized achievable rate for different transmitting schemes with different relay configuration. Simulations assumed that $N_1 = N_2 = N_{R_k} = 2$. For the proposed strategy, the maximized achievable rate is $R_i = R_j = 0.5\log_2(1 + \text{SNR}_{b,i}^{j,l,n_k})$. For scheme 1, the maximizing rate can be given as $R_{\text{max}} = 0.5\log_2\left(1 + \max_{j,l,n_k} \min_{i,k} \left(\text{SNR}_{b,i}^{j,l,n_k}, \text{SNR}_{b,j}^{j,l,n_k}\right)\right)$ owing to that the two sources have different received SNRs. For scheme 2, the maximizing transmitting rate of the two sources is also asymmetric, the maximizing rate of the system is $R_{\text{max}} = \frac{1}{2}\log_2(1 + \min_{j,k,n_k} \left(\text{SNR}_{b,j}^{j,l,n_k}, \text{SNR}_{b,k}^{j,l,n_k}\right))$, where $j, l, n_k$ are all random. For scheme 1 and scheme 2 the SNRs of the two sources can be denoted as follows.
In Fig. 5, it is observed that compared with scheme 1, the proposed scheme provides obvious rate gains. For example, with $P_T = 20 \text{dB}, K = 3$, the proposed scheme could provide 1.2 bit/s/Hz gains compared with scheme 1 using optimal power allocation and 2 bit/s/Hz gains compared with scheme 2 using joint relay-and-antenna selection and power allocation. Besides, it is observed that with the increase of $K$, the rate of scheme 1 and scheme 2 increase. While for verity of $K$, the rate of scheme 2 keeps constant which shows that rand selection scheme can’t get any performance gains.

6. Conclusions

In this paper, we developed a joint relay-and-antenna selection and optimal power allocation strategy for MIMO AF bidirectional relay networks consisting of two transceivers and multiple relay nodes which all implemented multiple antennas. Under the total power budget, we analyzed the optimal power allocation solution firstly, then with the optimal power
allocation solution we proposed a joint relay-and-antenna selection strategy based on DT technology with low complexity. Besides, a tight closed-form approximation of the overall outage probability was derived for the proposed strategy. Simulation results indicate that the proposed selection strategy improves the outage performance and the transmission rate greatly; also, the approximated outage probability expressions provide good approximations to the simulated results at almost all SNRs.

Appendix

In this appendix, we provide the derivation of the CDF for $Z_{n_{sx}}$.

The CDF of $X_{n_{sx}}$ and $Y_{n_{sy}}$ can be given by [22] as follows:

$$F_{X_{n_{sx}}}(x) = \sum_{p=0}^{N_1} \binom{N_1}{p} (-1)^p e^{-px}$$

(32)

$$F_{Y_{n_{sy}}}(y) = \sum_{q=0}^{N_2} \binom{N_2}{q} (-1)^q e^{-qy}$$

(33)

Moreover, $f_{X_{n_{sx}}}(x)$ and $f_{Y_{n_{sy}}}(y)$ are the PDFs of $X_{n_{sx}}$ and $Y_{n_{sy}}$ respectively, and can be given by:

$$f_{X_{n_{sx}}}(x) = \sum_{p=0}^{N_1-1} \binom{N_1}{p} (-1)^p e^{-(p+1)x}$$

(34)

$$f_{Y_{n_{sy}}}(y) = \sum_{q=0}^{N_2-1} \binom{N_2}{q} (-1)^q e^{-(q+1)y}$$

(35)

According to Equation (26) and (28), the CDF of $Z_{n_{sx}}$ can be expressed as follows:

$$F_{Z_{n_{sx}}}(z) \approx \Pr \left[ \frac{P_T^2 X_{n_{sx}} Y_{n_{sy}}}{2 \left( P_T^2 X_{n_{sx}} + P_T^2 Y_{n_{sy}} \right)^2} \leq z \right] = \Pr \left[ \frac{P_T X_{n_{sx}} Y_{n_{sy}}}{2 \left( X_{n_{sx}} + Y_{n_{sy}} \right)^2} \leq z \right]$$

(36)

We define $\sqrt{X_{n_{sx}}} = S$, $\sqrt{Y_{n_{sy}}} = T$, then
\[ F_{\beta_{\text{r,k}}} (z) = \Pr \left( \frac{P_{T} X_{n_{\text{r,k}}} Y_{n_{\text{r,k}}}}{2 \left( X_{n_{\text{r,k}}} + Y_{n_{\text{r,k}}} \right)} \leq z \right) = \Pr \left( \frac{P_{T}}{2 \left( \frac{1}{S} + \frac{1}{T} \right)} \leq z \right) \]  

(37)

Assuming \( S_{i} = 1/S \), \( T_{i} = 1/T \), then the formula above can be written as

\[ F_{\beta_{\text{r,k}}} (z) = \Pr \left[ \frac{1}{2 \left( S_{i} + T_{i} \right)} \leq \frac{z}{P_{T}} \right] = 1 - \int_{OAB} f_{S_{i}}(s_{i}) f_{T_{i}}(t_{i}) ds_{i} dt_{i} \]  

(38)

To reduce the computation complexity and get a closed-form expression, we enlarge slightly the integral region from \( OAB \) to \( OAD_{1}EB \) which is the sum of \( OAD_{1}D_{2} \) and \( D_{2}CEB \) as in Fig. 6. Then we can obtain

![Fig. 6. Integral region to determine overall outage probability](image)

Next, we derived the PDF and CDF of \( S_{i} \) as follows
\[ F_S(s) = \text{Pr}
\left( S \leq s \right) = \text{Pr}
\left( \sqrt{X_m} \leq s \right) = \text{Pr}
\left( X_m \leq s^2 \right) = F_{X_m}(s^2) \]  
(40)

\[ F_{s_i}(s_t) = \text{Pr}
\left( S_i \leq s_t \right) = \text{Pr}
\left( \frac{1}{S_i} \leq S_t \right) = 1 - F_{s_i}(\frac{1}{s_t}) = 1 - \frac{1}{s_t} \]  
(41)

\[ f_{s_i}(s_t) = -f_{s_t}
\left( \frac{1}{s_i^2} \right) \left( -2s_i^{-3} \right) = 2 \frac{1}{s_i^3} f_{s_t}
\left( \frac{1}{s_i^2} \right) = 2 \frac{1}{s_i^3} \sum_{p=0}^{N_i-1} \frac{N_i}{p} \left( -1 \right)^p e^{-\frac{(p+1)}{s_i}} \]  
(42)

Similarly, we can get the CDF and PDF of \( T_1 \) as

\[ F_{t_1}(t_1) = 1 - F_{y_n}(\frac{1}{t_1^2}) \]  
(43)

\[ f_{t_1}(t_1) = 2 \frac{N_i-1}{t_1^3} \sum_{q=0}^{N_i-1} \frac{N_i}{q} \left( -1 \right)^q e^{-\frac{(q+1)}{t_1}} \]  
(44)

Using (42) and (44), (39) can be calculated as follows

\[ F_{z_{n_i}}(z) \approx 1 - \left[ \int_{-\infty}^{\infty} 2s \sum_{p=0}^{N_i-1} \frac{N_i}{p} \left( -1 \right)^p e^{-\frac{(p+1)}{s^2}} \int_{-\infty}^{\infty} 2t \sum_{q=0}^{N_i-1} \frac{N_i}{q} \left( -1 \right)^q e^{-\frac{(q+1)}{t^2}} dtdst \right] 
+ \left[ \int_{-\infty}^{\infty} 2s \sum_{p=0}^{N_i-1} \frac{N_i}{p} \left( -1 \right)^p e^{-\frac{(p+1)}{s^2}} \int_{-\infty}^{\infty} 2t \sum_{q=0}^{N_i-1} \frac{N_i}{q} \left( -1 \right)^q e^{-\frac{(q+1)}{t^2}} dtdst \right] 
= 1 - 4 \sum_{p=0}^{N_i-1} \sum_{q=0}^{N_i-1} \frac{N_i}{p} \left( N_i - 1 \right) \left( -1 \right)^{p+q} 
\times \left[ \int_{-\infty}^{\infty} e^{-\frac{(p+1)x}{s^2}} ds \int_{-\infty}^{\infty} e^{-\frac{(q+1)y}{t^2}} dy \right] 
+ \left[ \int_{-\infty}^{\infty} e^{-\frac{(p+1)x}{s^2}} ds \int_{-\infty}^{\infty} e^{-\frac{(q+1)y}{t^2}} dy \right] 
= 1 - \sum_{p=0}^{N_i-1} \sum_{q=0}^{N_i-1} \frac{N_i}{p} \left( N_i - 1 \right) \left( -1 \right)^{p+q} 
\times \left( e^{-8(p+1)/t} + e^{-8(q+1)/s} \right) 
\]  
(45)
References


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