Joint Opportunistic Spectrum Access and Optimal Power Allocation Strategies for Full Duplex Single Secondary User MIMO Cognitive Radio Network

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Abstract

This paper introduces a full duplex single secondary user multiple-input multiple-output (FD-SSU-MIMO) cognitive radio network, where secondary user (SU) opportunistically accesses the authorized spectrum unoccupied by primary user (PU) and transmits data based on FD-MIMO mode. Then we study the network achievable average sum-rate maximization problem under sum transmit power budget constraint at SU communication nodes. In order to solve the trade-off problem between SU's sensing time and data transmission time based on opportunistic spectrum access (OSA) and the power allocation problem based on FD-MIMO transmit mode, we propose a simple trisection algorithm to obtain the optimal sensing time and apply an alternating optimization (AO)

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algorithm to tackle the FD-MIMO based network achievable sum-rate maximization problem. Simulation results show that our proposed sensing time optimization and AO-based optimal power allocation strategies obtain a higher achievable average sum-rate than sequential convex approximations for matrix-variable programming (SCAMP)-based power allocation for the FD transmission mode, as well as equal power allocation for the half duplex (HD) transmission mode.

Keywords: Full duplex, MIMO, opportunistic spectrum access, power allocation

1. Introduction

With the rapidly increasing spectrum requirements of emerging wireless communication service and application, cognitive radio (CR) is proposed to improve the spectrum utilization efficiency and solve the problem of congestion caused by traditional regular spectrum assignment. In cognitive radio network, opportunistic spectrum access (OSA), which is one of the most promising technologies to be implemented in dynamic spectrum access system as a replacement of static spectrum utilization rule, has a capability to access the spectrum holes according to prior primary spectrum sensing results. The basic idea of OSA is allowing the secondary user to identify the spectrum holes unoccupied by a primary user and access the authorized spectrum [1]. However, secondary user (SU) must vacate the spectrum holes once primary user returns back to access the channel again in order to protect the primary user from the harmful interference.

In order to satisfy the quality-of-service (QoS) requirements of SU and maximize the network achievable rate under the constraint that the primary user (PU) is sufficiently protected, a lots of previous works have studied on opportunistic spectrum sharing and power allocation strategies in cognitive radio network. The work in [2] proposed a joint power control and spectrum access scheme in CR network, which tackles the power allocation problem from the cooperative game perspective and solves the optimization problem of the proposed model with the differential evolution algorithm. In [3], the authors proposed continuous sensing-based power allocation strategies to maximize the achievable throughput of the SU in a multi-band CR network with perfect and quantized channel state information (CSI).

On the other hand, as a result of the requirement of high speed rate data transmission, multiple-input multiple-output (MIMO) communication techniques [4-5] have been paid considerable attention in recent years, because of the capability of greatly improving system reliability and spectral efficiency without more additional power. In [4], the authors considered the transmit optimization problem for a single secondary user MIMO and multiple-input single-output (MISO) channel in CR network under constraint of opportunistic spectrum sharing. In [5], the authors researched the joint beamforming and power allocation problem in cognitive MIMO systems via game theory in order to maximize the total throughput of secondary users. However, these works have focused on the spectrum access mode, power allocation strategies or MIMO.

Recently, the research on improving the spectral efficiency by the FD transmission mode has increased [6-7]. Obviously, comparing with half duplex (HD) transmission mode, the FD mode has the capability to greatly increase the communication system capacity, if the self-interference from the transmit antennas to the receive antennas at the same node can be efficiently eliminated [8-10]. Thus, the FD transmission mode has the potential to achieve more system sum-rate than the conventional HD transmission mode. However, the combination of power allocation and FD–MIMO in a CR network is not well-researched.

Motivated by these techniques, in this paper, we investigate joint opportunistic spectrum access and optimal power allocation strategies for the full duplex single secondary user MIMO (FD-SSU-MIMO) cognitive radio network. In our proposed network model, we pay much attention to how to solve the spectrum sensing time and data transmission time design problem and the power allocation problem of transmit antennas. In order to maximize the network achievable average sum-rate, we apply to a simple trisection algorithm to search the optimal spectrum sensing time, and then propose an alternating optimization (AO) algorithm to solve the power allocation optimization problem for the FD-SSU-MIMO cognitive radio network.

The rest of this paper is organized as follows. In Section 2, the FD-SSU-MIMO cognitive radio network model is introduced, and then the achievable average sum-rate maximization problem is formulated. In section 3, we study the trade-off problem between sensing time and data transmission time to maximize the average probability of spectrum holes discovery in the secondary network. And we propose AO-based optimal power allocation strategies applied to the FD-SSU-MIMO cognitive radio network in this section. Simulation results and discussions are presented in Section 4. Conclusions are drawn in Section 5.

The following notations are used in this paper. Bold upper case letter denotes matrix, bold low letter denotes vector, and non-bold letter denotes scalar. $\mathbf{G}^{H}$ represents the Hermitian transpose of matrix $\mathbf{G}$, $|\mathbf{G}|$ denotes the determinant of matrix $\mathbf{G}$, and $\mathrm{Tr}\{\mathbf{G}\}$ is the trace of matrix $\mathbf{G}$. $\mathrm{E}[\cdot]$ denotes the mathematical expectation operation. $\mathbf{I}_{m}$ represents the $m \times m$ unit matrix. $\mathbf{Q} \succeq 0$ indicates that $\mathbf{Q}$ is a positive semi-definite matrix.

2. Network Model and Problem Formulation

2.1 Network Model

We consider a FD-SSU-MIMO cognitive radio network, which is comprised of a pair of primary user transmitter (PU-Tx) and primary user receiver (PU-Rx), and two SU communication nodes as depicted in **Fig. 1**. Either of SU nodes is equipped with N_t transmit antennas and M_r receive antennas, which transmit and receive data respectively at the same time on the same frequency. SU can opportunistically access the primary channel when PU is detected to be absent. Once PU reoccupies the primary channel, SU must vacate the current channel and search a new available channel.

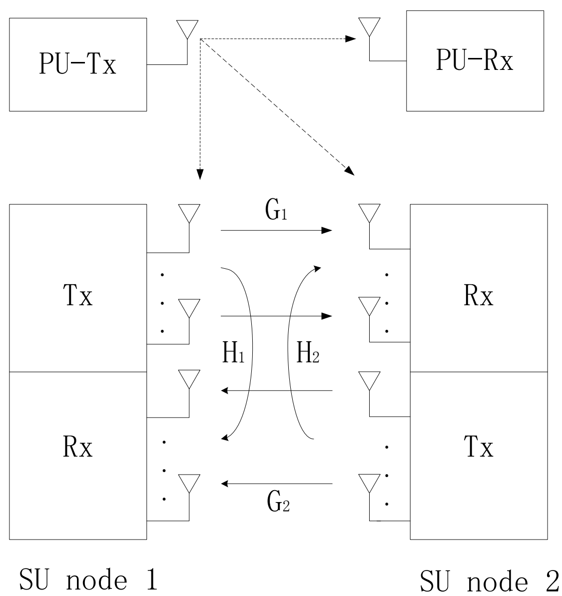


Fig. 1. FD-SSU-MIMO cognitive radio network model

In **Fig. 1**, we show the FD-SSU-MIMO cognitive radio network model, where $\mathbf{G}_j (j=1,2)$ denote the $M_r \times N_t$ channel power gain matrix from the one node's transmit antennas to the other node's receive antennas, and $\mathbf{H}_i (i=1,2)$ denote the $M_r \times N_t$ channel self-interference matrix from the i-th node's transmit antennas to the i-th node's receive antennas. $\mathbf{s}_i (i=1,2)$ is regarded as the $N_t \times 1$ transmitted signals vector of the i-th node. Let $\mathbf{P}_i$ be the $N_t \times N_t$ transmitted power matrix for the transmit antennas of the i-th node. Therefore, the expression for the received signal at the node 1 and node 2 are written as, respectively

$$\mathbf{z}_1 = \mathbf{G}_2\mathbf{P}_2\mathbf{s}_2 + \mathbf{H}_1\mathbf{P}_1\mathbf{s}_1 + \mathbf{w}_1 \tag{1a}$$

$$\mathbf{z}_2 = \mathbf{G}_1\mathbf{P}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{P}_2\mathbf{s}_2 + \mathbf{w}_2 \tag{1b}$$

where $\mathbf{w}_i\left(i=1,2\right)$ is the $M_r \times 1$ background noise at the i-th node which is assumed to be zero-mean complex Gaussian vector. The first part of (1) or (2) represents the received signals, and the second part represents the self-interference signals caused by the transmit antennas at the same node, which is treated as the background noise. Here, according to [7], we assume that $\mathrm{E}[\mathbf{s}_i\mathbf{s}_i^{\mathrm{H}}]=\mathbf{I}_{N_t}$ and $\mathrm{E}[\mathbf{s}_i\mathbf{s}_j^{\mathrm{H}}]=0\ (i \neq j)$. On the other hand, we suppose that the channel power gain matrix $\mathbf{G}_j\,(j=1,2)$ is known, and the self-interference channel matrix $\mathbf{H}_i\,(i=1,2)$ need to be estimated.

Let $\triangle_i$ be the estimated error matrix, and $\hat{\mathbf{H}}_i$ is the estimated channel matrix. Then, the actual self-interference channel matrix is given by

$$\mathbf{H}_i = \hat{\mathbf{H}}_i + \triangle_i \tag{2}$$

Let $\mathbf{\Sigma}_i$ denote $\mathrm{E}\left[\mathbf{w}_i\mathbf{w}_i^{\mathrm{H}}\right]$. From (1), we have

$$\begin{aligned}\mathrm{E}\left[\mathbf{z}_1\mathbf{z}_1^{\mathrm{H}}\right] &= \mathrm{E}\left[(\mathbf{G}_2\mathbf{P}_2\mathbf{s}_2+\mathbf{H}_1\mathbf{P}_1\mathbf{s}_1+\mathbf{w}_1)(\mathbf{G}_2\mathbf{P}_2\mathbf{s}_2+\mathbf{H}_1\mathbf{P}_1\mathbf{s}_1+\mathbf{w}_1)^{\mathrm{H}}\right]\\ &= \mathbf{G}_2\mathbf{P}_2(\mathbf{s}_2\mathbf{s}_2^{\mathrm{H}})\mathbf{P}_2^{\mathrm{H}}\mathbf{G}_2^{\mathrm{H}}+\mathbf{H}_1\mathbf{P}_1(\mathbf{s}_1\mathbf{s}_1^{\mathrm{H}})\mathbf{P}_1^{\mathrm{H}}\mathbf{H}_1^{\mathrm{H}}+\mathrm{E}\left[\mathbf{w}_1\mathbf{w}_1^{\mathrm{H}}\right]\\ &= \mathbf{G}_2\mathbf{P}_2\mathbf{P}_2^{\mathrm{H}}\mathbf{G}_2^{\mathrm{H}}+\mathbf{H}_1\mathbf{P}_1\mathbf{P}_1^{\mathrm{H}}\mathbf{H}_1^{\mathrm{H}}+\mathbf{\Sigma}_1\end{aligned} \tag{3a}$$

$$\begin{aligned}\mathrm{E}\left[\mathbf{z}_2\mathbf{z}_2^{\mathrm{H}}\right] &= \mathrm{E}\left[(\mathbf{G}_1\mathbf{P}_1\mathbf{s}_1+\mathbf{H}_2\mathbf{P}_2\mathbf{s}_2+\mathbf{w}_2)(\mathbf{G}_1\mathbf{P}_1\mathbf{s}_1+\mathbf{H}_2\mathbf{P}_2\mathbf{s}_2+\mathbf{w}_2)^{\mathrm{H}}\right]\\ &= \mathbf{G}_1\mathbf{P}_1(\mathbf{s}_1\mathbf{s}_1^{\mathrm{H}})\mathbf{P}_1^{\mathrm{H}}\mathbf{G}_1^{\mathrm{H}}+\mathbf{H}_2\mathbf{P}_2(\mathbf{s}_2\mathbf{s}_2^{\mathrm{H}})\mathbf{P}_2^{\mathrm{H}}\mathbf{H}_2^{\mathrm{H}}+\mathrm{E}\left[\mathbf{w}_2\mathbf{w}_2^{\mathrm{H}}\right]\\ &= \mathbf{G}_1\mathbf{P}_1\mathbf{P}_1^{\mathrm{H}}\mathrm{G}_1^{\mathrm{H}}+\mathbf{H}_2\mathbf{P}_2\mathbf{P}_2^{\mathrm{H}}\mathbf{H}_2^{\mathrm{H}}+\mathbf{\Sigma}_2\end{aligned} \tag{3b}$$

The achievable rate at the node 1 and node 2 are:

$$\begin{aligned}R_1 &= \log_2\left|\mathbf{I}_{M_r}+\left(\mathbf{\Sigma}_1+\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^{\mathrm{H}}\right)^{-1}\mathbf{G}_2\mathbf{Q}_2\mathbf{G}_2^{\mathrm{H}}\right|\\ &= \log_2\left|\mathbf{\Sigma}_1+\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^{\mathrm{H}}+\mathbf{G}_2\mathbf{Q}_2\mathbf{G}_2^{\mathrm{H}}\right|-\log_2\left|\mathbf{\Sigma}_1+\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^{\mathrm{H}}\right|\end{aligned} \tag{4a}$$

$$\begin{aligned}R_2 &= \log_2\left|\mathbf{I}_{M_r}+\left(\mathbf{\Sigma}_2+\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^{\mathrm{H}}\right)^{-1}\mathbf{G}_1\mathbf{Q}_1\mathbf{G}_1^{\mathrm{H}}\right|\\ &= \log_2\left|\mathbf{\Sigma}_2+\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^{\mathrm{H}}+\mathbf{G}_1\mathbf{Q}_1\mathbf{G}_1^{\mathrm{H}}\right|-\log_2\left|\mathbf{\Sigma}_2+\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^{\mathrm{H}}\right|\end{aligned} \tag{4b}$$

where $\mathbf{Q}_i=\mathbf{P}_i\mathbf{P}_i^{\mathrm{H}}\,(i=1,2)$ represents the transmit power covariance matrix at the i-th node. Then, the FD sum-rate is

$$R_{sum} = R_1 + R_2 \tag{5}$$

2.2 Spectrum Sensing and Data Transmission Design

In this work, we assume that the CR network operates on frame structure of fixed duration. The duration of each frame consists of two slots: sensing slot τ and data transmission slot $T-\tau$, as shown in **Fig. 2**. Then the SU carries out periodic spectrum sensing to decide whether the PU is absent or not. In OSA mode, the SU must frequently sense the spectrum before accessing the licensed spectrum. The spectrum holes appear only when the PU are detected to be not busy, for the sake of protecting the PU from the harmful interference.

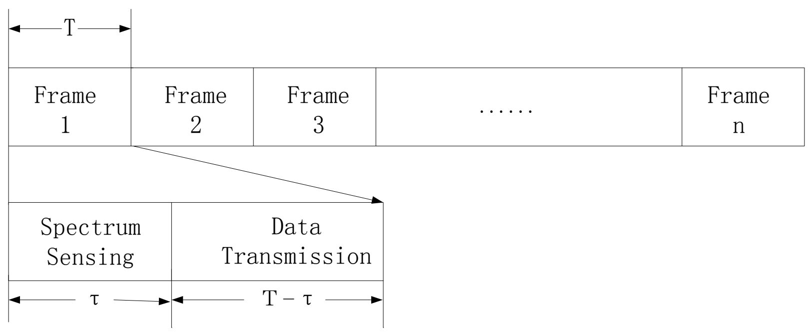


Fig. 2. Frame structure design for the CR network

Let H_0 and H_1 be two hypotheses that the PU is absent and the PU is present, respectively. In the single threshold based energy detection method, the final decision result depend on the predefined threshold λ_{th}, shown as

$$\text{final decision}=\begin{cases}\text{PU is absence, if } E_s \le \lambda_{th} \\ \text{PU is presence, if } E_s > \lambda_{th}\end{cases} \tag{6}$$

where E_s denotes the energy of the received sample signal. CR makes a final decision whether PU is absent or not in accordance with the sample signal energy E_s and the predefined threshold λ_{th}. Usually, two metrics are used to evaluate the detection performance: the false alarm probability P_f and the detection probability P_d.

According to the central limit theorem, the sample signal statistic can be approximated by a Gaussian distribution when the sample number is large enough. Let f_s stand for the sample frequency. σ_n^2 denotes the variance of Gaussian noise and γ represents the received PU signal to noise ratio. Thus, in the energy detection method, the P_f and P_d are derived as [11]

$$P_f = P(E_s > \lambda_{th} | H_0) = Q\left(\left(\frac{\lambda_{th}}{\sigma_n^2} - 1\right)\sqrt{\tau f_s}\right) \tag{7}$$

$$P_d = P(E_s > \lambda_{th} | H_1) = Q\left(\left(\frac{\lambda_{th}}{\sigma_n^2} - 1 - \gamma\right)\sqrt{\frac{\tau f_s}{2\gamma+1}}\right) \tag{8}$$

where $Q(.)$ denotes the Q-function defined as $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{t^2}{2}}dt$.

Let $P(H_0)$ and $P(H_1)$ be the probability that PU is absent and the probability that PU is present, respectively. Then, the probability of spectrum holes discovery in OSA mode is shown as

$$P_{free}=(1-P_f)P(H_0)+(1-P_d)P(H_1) \tag{9}$$

where $(1-P_f)P(H_0)$ indicates the probability that the PU is idle and SU make a right decision, and $(1-P_d)P(H_1)$ represents the probability that the PU is busy but SU do not detect accurately.

2.3 Problem Formulation

In this paper, we are interested in maximizing the achievable average FD sum-rate under the sum transmit power budget constraint of SU node. The achievable average FD sum-rate in the OSA mode can be given by

$$R_{OSA}=\frac{T\text{ - }\tau}{T}((1-P_f)P(H_0)+(1-P_d)P(H_1))(R_1+R_2) \tag{10}$$

Therefore, this problem can be formulated as

$$\begin{aligned} &\max_{\tau,\mathbf{Q_1},\mathbf{Q_2}} \quad R_{OSA} \\ &\text{subject to}: P_f \le P_{f,tar} \\ &\qquad P_d \ge P_{d,tar} \\ &\qquad 0<\tau<T \\ &\qquad \text{Tr}\{\mathbf{Q}_i\} \le P_{max} \\ &\qquad \mathbf{Q}_i \succeq 0, i=1,2 \end{aligned} \tag{11}$$

where the positive semi-definite constraint conditions guarantee that the transmit power covariance matrices are feasible. P_{max} stands for the total transmit power budget of SU node. $P_{d,tar}$ and $P_{f,tar}$ are the target detection probability and the target false alarm probability on condition that the PU is sufficiently protected, respectively. Usually, in order to improve the unoccupied spectrum utilization and reduce the interference to PU, they satisfy $P_{d,tar} \ge 0.9$ and $P_{f,tar} \le 0.1$. It is pointed out that if the primary user requires 100% protection in its authorized spectrum, the secondary user is not allowed to access the authorized spectrum in OSA mode because it is not guaranteed that the detection probability P_d is equal to 1. However, since the target detection probability $P_{d,tar}$ is more than 0.9 and $P_d \ge P_{d,tar}$, the probability $(1-P_d)P(H_1)$ of producing the harmful interference to PU is very small and acceptable.

According to (7) and (8), it is obvious that (9) is related with the variable τ and independent of $\mathbf{Q}_i$. However, $R_i(i=1,2)$ is independent of τ and is related with $\mathbf{Q}_i$. Thus, the maximization problem (11) can be divided into two sub-problems. Then (9) is equivalent to (12) and (13):

$$\begin{aligned} \max_{\tau} \quad & \frac{T-\tau}{T}((1-P_f)P(H_0)+(1-P_d)P(H_1)) \\ \text{subject to:} \quad & P_f \le P_{f,tar} \\ & P_d \ge P_{d,tar} \\ & 0<\tau<T \end{aligned} \tag{12}$$

$$\begin{aligned} \max_{\mathbf{Q}_1,\mathbf{Q}_2} \quad & R_{sum} = R_1 + R_2 \\ \text{subject to:} \quad & \mathrm{Tr}\{\mathbf{Q}_i\} \le P_{max} \\ & \mathbf{Q}_i \succeq 0, i=1,2 \end{aligned} \tag{13}$$

In the next section, we will solve the above optimization problem (12) and (13), respectively.

3. Optimal Sensing Time Design and Optimal Power Allocation Strategies

3.1 Optimal Sensing Time Design

In the previous section, the relationship between sensing time and the achievable average FD sum-rate in the OSA mode has been derived. In this section, we will design the sensing time and data transmission time to maximize the achievable average FD sum-rate of the cognitive radio network. In OSA mode, SU need to perform spectrum sensing so that it could find spectrum holes and access the unused licensed spectrum without the harmful interference to PU. For a fixed frame duration T, the longer the spectrum sensing time τ, the shorter the data transmission time $T-\tau$. The longer spectrum sensing time causes much overhead and mitigates data transmission time of SU, while short sensing time makes it difficulty to guarantee the acceptable detection probability and false alarm probability requirement. Therefore, it is necessary to consider the trade-off between the spectrum sensing time and data transmission time to find the optimal sensing time τ in order to achieve the maximal sum-rate while PU is sufficiently protected.

Next, we will demonstrate the existence of the optimal sensing time to obtain the object function maximal value of (12). Let $F(\tau)$ represent the average probability of spectrum holes discovery.

$$F(\tau)=\frac{T-\tau}{T}((1-P_f)P(H_0)+(1-P_d)P(H_1)) \tag{14}$$

Thus, (12) is equivalent to

$$\begin{aligned} &\max_{\tau} \quad F(\tau) \\ &\text{subject to:} \quad P_f \le P_{f,tar} \\ &\qquad P_d \ge P_{d,tar} \\ &\qquad 0 < \tau < T \end{aligned} \tag{15}$$

From (7) and (8), for a given target detection probability $P_{d,tar}$ and a given target false alarm probability $P_{f,tar}$, we have $P_f = Q(\alpha)$ and $P_d = Q(\beta)$, where $\alpha = Q^{-1}(P_{d,tar})\sqrt{2\gamma+1} + \gamma\sqrt{\tau f_s}$ and $\beta = \frac{\left(Q^{-1}(P_{f,tar}) - \gamma\sqrt{\tau f_s}\right)}{2\gamma+1}$. According to literature [12], P_d is an increasing and concave function of τ under $P_d > 0.5$ and P_f is a decreasing and convex function of τ under $P_f < 0.5$. Thus, we have $\alpha > 0$ and $\beta < 0$. By using $Q(-x) = 1 - Q(x), x > 0$, the $Q(x)$ is approximately equal to [13]

$$Q(x) = \begin{cases} \dfrac{1 - e^{-\frac{C_1}{\sqrt{2}}x}}{C_2\sqrt{2\pi}x} e^{-\frac{x^2}{2}}, & x > 0 \\ 1 + \dfrac{1 - e^{\frac{C_1}{\sqrt{2}}x}}{C_2\sqrt{2\pi}x} e^{-\frac{x^2}{2}}, & x \le 0 \end{cases} \tag{16}$$

where $C_1 = 1.98$ and $C_2 = 1.135$.

Furthermore, by using (16), we have

$$P_f = Q(\alpha) = \frac{1 - e^{-\frac{C_1}{\sqrt{2}}\alpha}}{C_2\sqrt{2\pi}\alpha} e^{-\frac{\alpha^2}{2}}, \alpha > 0 \tag{17}$$

$$P_d = Q(\beta) = 1 + \frac{1 - e^{\frac{C_1}{\sqrt{2}}\beta}}{C_2\sqrt{2\pi}\beta} e^{-\frac{\beta^2}{2}}, \beta < 0 \tag{18}$$

Then, we will prove that there indeed exists a maximum value $F(\tau)$ about τ within the interval $(0, T)$.

Proof: Differentiating (14) with respect to τ, we have

$$\frac{F(\tau)}{\partial\tau}= -\frac{1}{T}\Big(\big(1-Q(\alpha)\big)P(H_0)+(1-Q(\beta))P(H_1)\Big)$$
$$+\frac{T-\tau}{T}\left(\frac{P(H_0)\gamma\sqrt{f_s}}{2\sqrt{\tau}}\cdot\frac{-C_1\alpha e^{-\frac{C_1}{\sqrt{2}}\alpha}+\sqrt{2}\left(1+\alpha^2\right)\left(1-e^{-\frac{C_1}{\sqrt{2}}\alpha}\right)}{2C_2\sqrt{\pi}\alpha^2}\cdot e^{-\frac{\alpha^2}{2}}\right.$$
$$\left.+\frac{P(H_1)\gamma\sqrt{f_s}}{2(2\gamma+1)\sqrt{\tau}}\cdot\frac{-C_1\beta e^{\frac{C_1}{\sqrt{2}}\beta}-\sqrt{2}\left(1+\beta^2\right)\left(1-e^{\frac{C_1}{\sqrt{2}}\beta}\right)}{2C_2\sqrt{\pi}\beta^2}\cdot e^{-\frac{\beta^2}{2}}\right) \tag{19}$$

Obviously, as a result of fact that the lower bound of $Q(x)$ is 0 and the upper bound is 1, we have

$$\lim_{\tau\to T}\frac{F(\tau)}{\partial\tau}=-\lim_{\tau\to T}\frac{1}{T}\Big(\big(1-Q(\alpha)\big)P(H_0)+(1-Q(\beta))P(H_1)\Big) < 0 \tag{20}$$

$$\begin{aligned}\lim_{\tau\to 0}\frac{F(\tau)}{\partial\tau}&=-\lim_{\tau\to 0}\frac{1}{T}\Big(\big(1-Q(\alpha)\big)P(H_0)+(1-Q(\beta))P(H_1)\Big)\\&\quad+\lim_{\tau\to 0}\frac{P(H_0)\gamma\sqrt{f_s}}{2\sqrt{\tau}}\cdot\frac{-C_1\alpha e^{-\frac{C_1}{\sqrt{2}}\alpha}+\sqrt{2}\left(1+\alpha^2\right)\left(1-e^{-\frac{C_1}{\sqrt{2}}\alpha}\right)}{2C_2\sqrt{\pi}\alpha^2}\cdot e^{-\frac{\alpha^2}{2}}\\&\quad+\lim_{\tau\to 0}\frac{P(H_1)\gamma\sqrt{f_s}}{2(2\gamma+1)\sqrt{\tau}}\cdot\frac{-C_1\beta e^{\frac{C_1}{\sqrt{2}}\beta}-\sqrt{2}\left(1+\beta^2\right)\left(1-e^{\frac{C_1}{\sqrt{2}}\beta}\right)}{2C_2\sqrt{\pi}\beta^2}\cdot e^{-\frac{\beta^2}{2}}\\&=+\infty\end{aligned} \tag{21}$$

Proof of (21): See Appendix A.

According to the zero theorem, there exists a value τ_0 within $(0,T)$ at least to satisfy $\frac{F(\tau)}{\partial\tau}\Big|_{\tau=\tau_0}=0$, because of $\frac{F(\tau)}{\partial\tau}$ is a continuous differential function of variable τ. It means that $F(\tau)$ is a increasing function for the smaller τ, and it becomes a decreasing function when τ approach to T. Thus, there exists a maximal value of $F(\tau)$ within $(0,T)$.

As a result of not obtaining the optimal sensing time τ in a closed form expression from (15), we will adopt a simple trisection Algorithm to search the optimal τ that make

$F(\tau)$ acquire the maximal value, as shown in the following Algorithm 1.

Table 1. Algorithm 1

```
Initialization: set τ_min=0  and τ_max=T
Define eps=0.01
While(τ_max - τ_min< eps)
   do      τ_mid1=(τ_min+τ_max)/2      and
           τ_mid2=(τ_mid1+τ_max)/2
   compute       τ_mid1_F=F(τ_mid1)     and
                τ_mid2_F=F(τ_mid2)
           if (τ_mid1_F>=τ_mid2_F)
                 τ_max=τ_mid2
           end
           else
                 τ_min=τ_mid1
           end
End
Output     τ_optimal=(τ_mid+τ_max)/2
```

3.2 Optimal Power Allocation Strategies

Despite of the many previous literates on power allocation strategies in wireless communication network, the power allocation problem of transmit antennas about the FD-SSU-MIMO cognitive radio network under total transmit power constraints is not well-studied. Therefore, in this paper, we consider the power allocation strategies applied to the FD-SSU-MIMO cognitive radio network in order to maximize the achievable average FD sum-rate. As described in previous section, in order to reduce the effect of the self-interference for FD transmission mode, it is necessary to optimize the transmit antennas power allocation at each node under the node total transmit power constraints.

From the maximization problem (13), we can obtain the follow equivalent problem

$$\begin{aligned} &\max_{\mathbf{Q}_1,\ \mathbf{Q}_2} \quad f_1 + f_2 + g_1 + g_2 \\ &\text{subject to: } \operatorname{Tr}\{\mathbf{Q}_i\} \le P_{max} \\ &\qquad\qquad \mathbf{Q}_i \succeq 0, i = 1, 2 \end{aligned} \tag{22}$$

where f_i and g_i are represented by

$$f_i = \log_2 \left| \mathbf{\Sigma}_i + \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^{\mathrm{H}} + \mathbf{G}_j \mathbf{Q}_j \mathbf{G}_j^{\mathrm{H}} \right| \tag{23}$$

$$g_i = -\log_2 \left| \mathbf{\Sigma}_i + \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^{\mathrm{H}} \right| \tag{24}$$

Obviously, f_i is concave and g_i is non-concave. Thus, the maximization optimization problem (23) is non-concave and difficult to solve directly. In this paper, we will apply an alternating optimization (AO) algorithm [14] to solve the non-concave optimization problem (22). Before describing the AO algorithm, we need to introduce a lemma [15].

Lemma 1: Let $\mathbf{E}$ be any $m \times m$ matrix such that $\mathbf{E} \succ 0$ and $|\mathbf{E}| \le 1$. Consider the function $h(\mathbf{S}) = -\mathrm{Tr}\{\mathbf{SE}\} + \log_2 |\mathbf{S}| + m$, where $\mathbf{S}$ is the $m \times m$ matrix. Then,

$$\max_{\mathbf{S} \succeq 0} h(\mathbf{S}) = \log_2 \left| \mathbf{E}^{-1} \right| \tag{25}$$

with the optimum value $\mathbf{S}_{\mathrm{opt}} = \mathbf{E}^{-1}$.

Let $\mathbf{E} = \mathbf{\Sigma}_i + \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^{\mathrm{H}}$, where $\mathbf{E}$ is the $m \times m$ matrix, by applying lemma 1 to (24), we have

$$\hat{g}_i = -\mathrm{Tr}\left\{ \mathbf{S}_i (\mathbf{\Sigma}_i + \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^{\mathrm{H}}) \right\} + \log_2 |\mathbf{S}_i| + M_r \tag{26}$$

$$\mathbf{S}_i = (\mathbf{\Sigma}_i + \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^{\mathrm{H}})^{-1} \tag{27}$$

where (26) is a concave function. Thus, equivalent formulation of problem (22) is given by

$$\begin{aligned} \max_{\mathrm{Q}_1, \mathrm{Q}_2} \quad & f_1 + f_2 + \hat{g}_1 + \hat{g}_2 \\ \text{subject to:} \quad & \mathrm{Tr}\{\mathbf{Q}_i\} \le P_{max} \\ & \mathbf{Q}_i \succeq 0, i = 1, 2 \\ & \mathbf{S}_i \succeq 0, i = 1, 2 \end{aligned} \tag{28}$$

The objective function of (28) is concave and equivalent to the original objective function of (22). The AO-based Algorithm solves the approximate concave programming problem by iteratively updating the objective function of (28) until convergence by using CVX package in Matlab, as described in the following Algorithm 2.

Table 2. Algorithm 2

Initialization: randomly set Q_1 , Q_2, $Sum_0=R_1+R_2$, $Sum_1=1$; **Define** eps=0.0001; **While** | Sum_1- Sum_0 |>eps $Sum_0=Sum_1$; update S_1 , S_2 by (28); update $\hat{g}_1, \hat{g}_2$ by (27); update $f_1 + f_2 + \hat{g}_1 + \hat{g}_2$; solve Q_1^{opt}, Q_2^{opt} by CVX; update $Q_1 = Q_1^{(opt)}, Q_2 = Q_2^{(opt)}$; update Sum_1 by (5); **End** **Output** Q_1^{opt}, Q_2^{opt} , $R_{sum}=Sum_1$; **Solve** P_i by using Cholesky Decomposition: $Q_i = P_i P_i^H \ (i = 1, 2)$.

According to Algorithm 1 and Algorithm 2, the achievable average sum-rate in FD-SSU-MIMO network in OSA mode is obtained by

$$R_{OSA} = F(\tau\_optimal) R_{sum} \tag{29}$$

4. Simulation Results and Discussion

In this section, we provide simulation examples to evaluate the network performance of the proposed sensing time optimization and optimal power allocation strategies. In spectrum sensing simulation process, we assume that the target detection probability $P_{d,tar} = 0.95$, the target false alarm probability $P_{f,tar} = 0.05$, the fixed time duration $T = 100\text{ms}$, the sample frequency $f_s = 10\text{KHz}$.

Fig. 3 illustrates the relationship between the average probability of spectrum holes discovery $F(\tau)$ and sensing time τ under different $P(H_0)$. Obviously, from the **Fig. 3**, the average probability of spectrum holes discovery $F(\tau)$ indeed exist a maximum value. For example, the maximal value of $F(\tau)$ is about 0.71 under $P(H_0) = 0.7$.

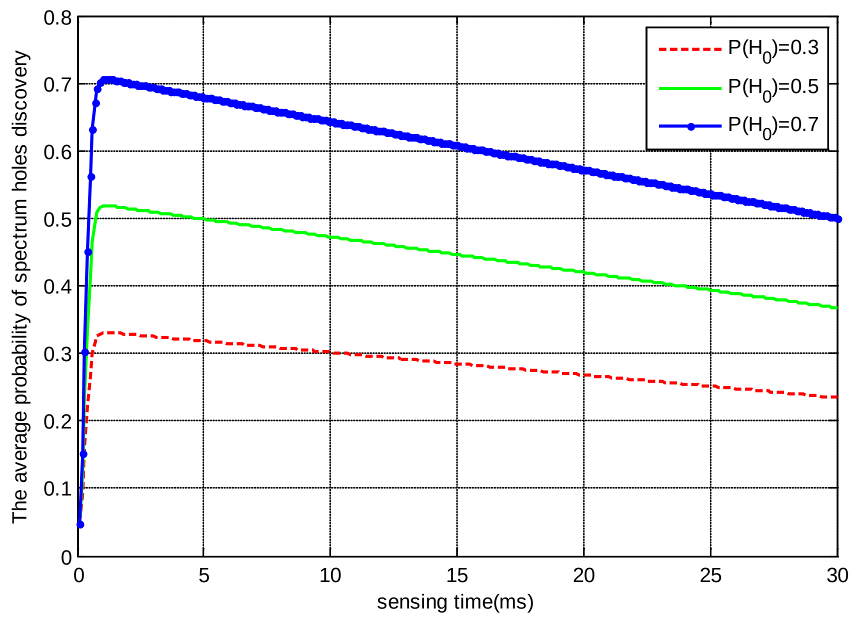


Fig. 3. $F(\tau)$ vs. sensing time τ with $T = 100\text{ms}$

Then in data transmission simulation process, the noise covariance matrix $\mathbf{\Sigma}_i$ is normalized to unit matrix. In [7], the authors point out that no standard reference self-interference channel model has been reported, and self-interference channel matrix simply is generated as a zero-mean complex Gaussian random variable. In [16], it is assumed that the true self-interference channel matrix consists of the estimated channel matrix and the channel estimation error matrix, which are generated as a zero-mean complex Gaussian random variable. In this paper, we assume that the channel power gain matrix $\mathbf{G}_j$, and the channel self-interference gain matrix $\hat{\mathbf{H}}_i$ are zero-mean complex Gaussian random variables, and variance are equal to the signal to noise ratio (SNR) and the self-interference to noise ratio (INR), respectively. Furthermore, we suppose that the estimated error matrix $\triangle_i$ is also zero-mean complex Gaussian random variable with variance equal to σ_e^2. Let SIR represent the ratio of SNR and INR. The transmit power budget P_{max} of two nodes are identical. The number of transmit antennas N_t and receive antennas M_r are identical.

Fig. 4 compares the network achievable average sum-rate R_{OSA} versus SNR under conventional equal power allocation for HD, SCAMP-based power allocation for FD proposed in [16], and our proposed AO-based power allocation for FD with the fixed $SIR = \text{-10dB}$, $P(H_0)\text{=}0.7$, $\sigma_e^2\text{=}1$ and $N_t = M_r = 4$. From **Fig. 4**, as the SNR is increased, our proposed AO-based power allocation for FD obtains more increment than conventional equal power allocation for HD and SCAMP-based power allocation for FD in the network achievable average sum-rate. It is obvious that conventional equal power allocation for HD is not optimal scheme under the constraint of the total transmit power because it fails to optimize the transmit antenna power and use the FD transmission mode. For example, our proposed AO-based power allocation for FD obtains more 1.5 (bit/s/Hz) increment than conventional equal power allocation for HD when the SNR is equal to 0 (dB), and more 2.5 (bit/s/Hz) increment than conventional equal power allocation for HD when the SNR is equal to 10 (dB). On the other hand, for a fixed SNR, our proposed AO-based power allocation for FD obtains about 1(bit/s/Hz) increment than SCAMP-based power allocation for FD. The results indicate that our proposed AO-based power allocation for FD obtain higher performance improvement than conventional equal power allocation for HD and SCAMP-based power allocation for FD.

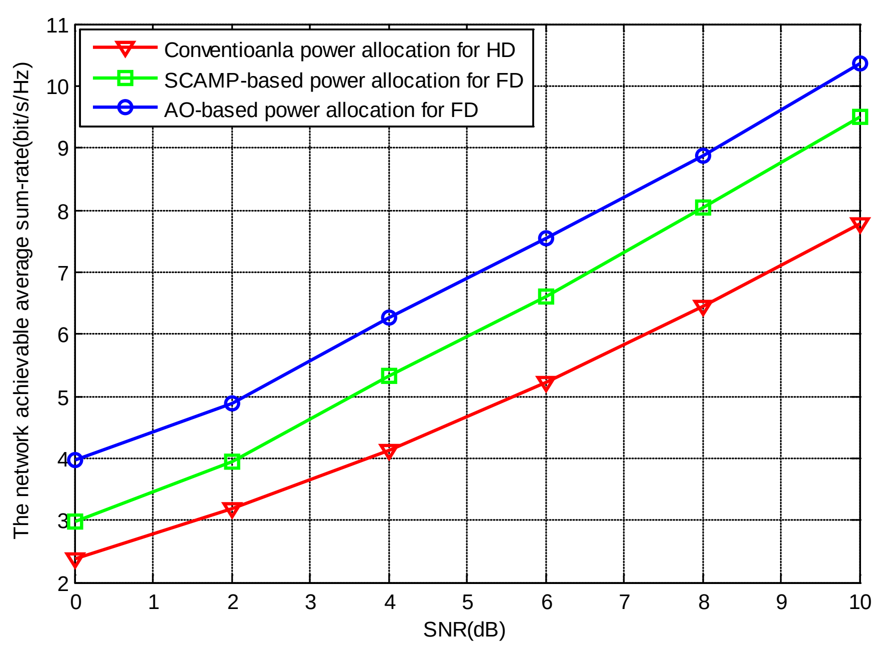


Fig. 4. R_{OSA} vs. SNR with $N_t = M_r = 4$, SIR=-10dB, $P_{max} = 1$w, σ_e^2=1

Fig. 5 shows the network achievable average sum-rate R_{OSA} versus the total transmit power budget P_{max} of SU node under conventional equal power allocation for HD, SCAMP-based power allocation for FD, and our proposed AO-based power allocation for FD with the fixed SIR = -10dB, $P(H_0)$=0.7, σ_e^2=1, SNR = 10dB and $N_t = M_r = 4$. Obviously, the results indicate that our proposed AO-based power allocation for FD obtain about 30-50% performance improvement compared with conventional equal power allocation for HD, as well as about 5-20% performance improvement compared with SCAMP-based power allocation for FD. Therefore, our proposed AO-based power allocation provides the best average sum-rate.

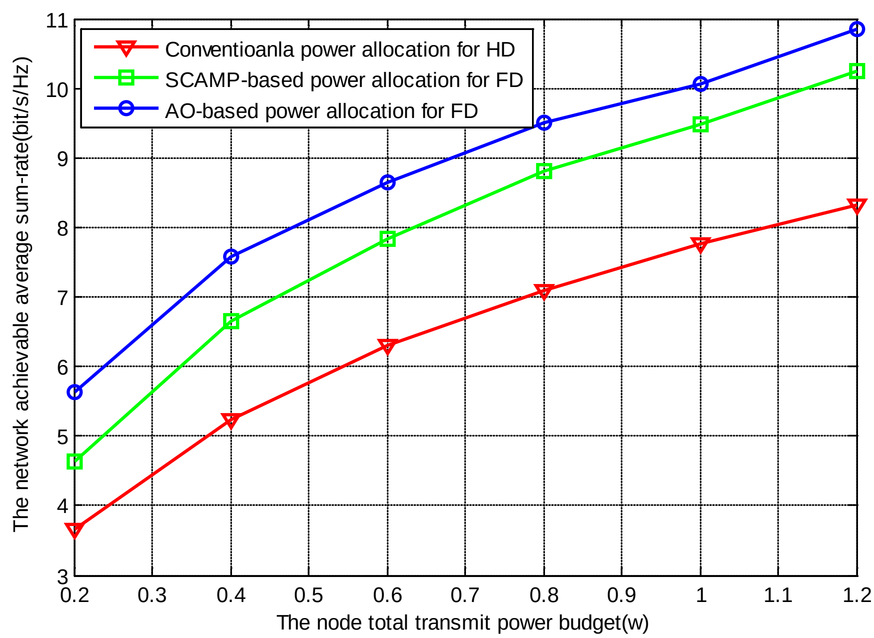


Fig. 5. R_{OSA} vs. P_{max} with $N_t = M_r = 4$, SIR=-10dB, SNR=10dB, σ_e^2=1

Fig. 6 shows the comparisons of the network achievable average sum-rate R_{OSA} with the number of different antennas under our proposed AO-based power allocation for FD, SCAMP-based power allocation for FD and conventional equal power allocation for HD. As seen from **Fig. 6,** when the number of antennas increases, the network achievable average sum-rate increases. Clearly, the results show that our proposed AO-based power allocation for FD obtain about 20-30% performance improvement compared with conventional equal power allocation for HD, as well as about 5-20% performance improvement compared with SCAMP-based power allocation for FD. Thus, the results indicate our proposed power allocation algorithm achieves the better performance improvement for the different antennas number.

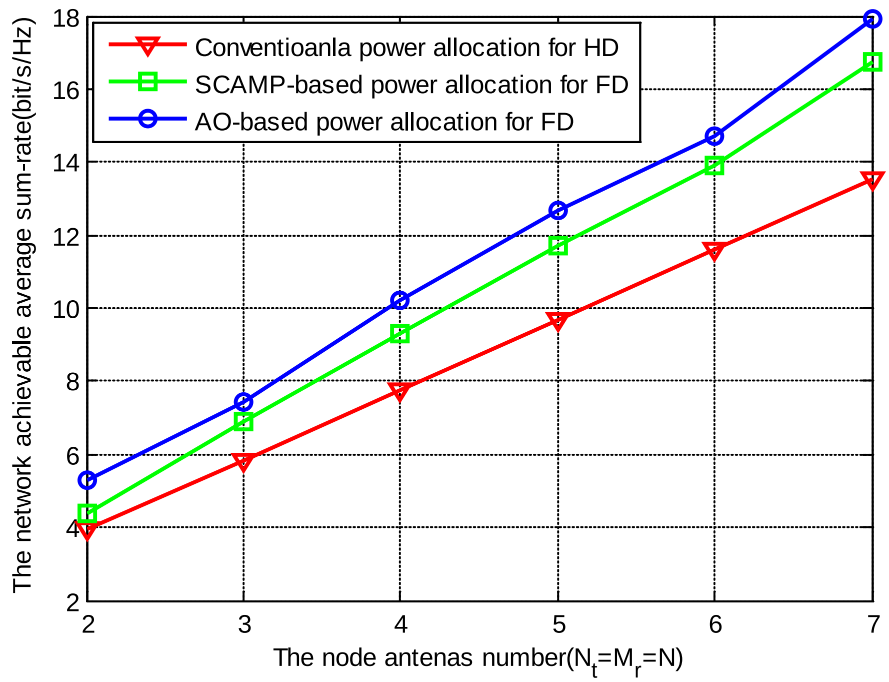


Fig. 6. R_{OSA} vs. N with SIR=-10dB, SNR=10dB, $P_{max}=1$w, $\sigma_e^2=1$

5. Conclusions

In this paper, we have introduced a FD-SSU-MIMO cognitive radio network, where SU can transmit and receive data respectively at the same time on the same primary frequency when PU is detected to be absent. We research the CR network frame structure design of spectrum sensing time and data transmission time in OSA to find the maximal average probability of spectrum holes discovery. And then we propose optimal power allocation strategies for the FD-SSU-MIMO cognitive radio network in order to maximizing the average achievable sum-rate of secondary user network.

Simulation results demonstrate that our proposed joint sensing time optimization and optimal power allocation strategies can achieve a higher average achievable sum-rate than conventional equal power allocation for HD transmission mode, as well as SCAMP-based power allocation for FD transmission mode.

Appendix A

This appendix proves (21).

Proof: Let

$$f_1(\alpha) = C_1 \alpha e^{-\frac{C_1}{\sqrt{2}}\alpha} + \sqrt{2}\left(1+\alpha^2\right)\left(1-e^{-\frac{C_1}{\sqrt{2}}\alpha}\right) \tag{A.1}$$

$$f_2(\beta) = C_1 \beta e^{\frac{C_1}{\sqrt{2}}\beta} - \sqrt{2}\left(1+\beta^2\right)\left(1-e^{\frac{C_1}{\sqrt{2}}\beta}\right) \tag{A.2}$$

where $\alpha > 0, \beta < 0, C_1 = 1.98, C_2 = 1.135$.

Differentiating (A.1) with respect to α and (A.2) with respect to β, respectively, we have

$$\frac{df_1(\alpha)}{d\alpha} = \frac{C_1^2 \alpha}{\sqrt{2}} e^{-\frac{C_1}{\sqrt{2}}\alpha} + 2\sqrt{2}\alpha\left(1-e^{-\frac{C_1}{\sqrt{2}}\alpha}\right) + C_1 \alpha^2 e^{-\frac{C_1}{\sqrt{2}}\alpha} \tag{A.3}$$

$$\frac{df_2(\beta)}{d\beta} = -\frac{C_1^2 \beta}{\sqrt{2}} e^{\frac{C_1}{\sqrt{2}}\beta} - 2\sqrt{2}\beta\left(1-e^{\frac{C_1}{\sqrt{2}}\beta}\right) + C_1 \beta^2 e^{\frac{C_1}{\sqrt{2}}\beta} \tag{A.4}$$

For $\alpha > 0$ and $\beta < 0$, it is obvious that $\frac{df_1(\alpha)}{d\alpha} > 0$ and $\frac{df_1(\beta)}{d\beta} > 0$ .Furthermore, from (A.1) and (A.2), notice $f_1(0) = 0$ and $f_2(0) = 0$. Thus, we have

$$f_1(\alpha) > 0 \tag{A.5}$$

$$f_1(\beta) > 0 \tag{A.6}$$

Therefore,

$$\begin{aligned}
\lim_{\tau\to 0}\frac{F(\tau)}{\partial\tau} &= -\lim_{\tau\to 0}\frac{1}{T}\Big(\big(1-Q(\alpha)\big)P(H_0) + (1-Q(\beta))P(H_1)\Big) \\
&+ \lim_{\tau\to 0}\frac{P(H_0)\gamma\sqrt{f_s}}{2\sqrt{\tau}} \cdot \frac{-C_1\alpha e^{-\frac{C_1}{\sqrt{2}}\alpha} + \sqrt{2}\left(1+\alpha^2\right)\left(1-e^{-\frac{C_1}{\sqrt{2}}\alpha}\right)}{2C_2\sqrt{\pi}\alpha^2} \cdot e^{-\frac{\alpha^2}{2}} \\
&+ \lim_{\tau\to 0}\frac{P(H_1)\gamma\sqrt{f_s}}{2(2\gamma+1)\sqrt{\tau}} \cdot \frac{-C_1\beta e^{\frac{C_1}{\sqrt{2}}\beta} - \sqrt{2}\left(1+\beta^2\right)\left(1-e^{\frac{C_1}{\sqrt{2}}\beta}\right)}{2C_2\sqrt{\pi}\beta^2} \cdot e^{-\frac{\beta^2}{2}} \\
&= +\infty
\end{aligned} \tag{A.7}$$

References

[1] X. Zhou, G.Y. Li, D. Li, D. Wang and A.C. Soong, "Probabilistic resource allocation for opportunistic spectrum access," *IEEE Transactions on Wireless Communications*, vol. 9, no. 9, pp. 2870-2879, September, 2010. Article (CrossRef Link).

[2] Q. Song, Z. Ning, Y. Huang, L. Guo and X. Lu, "Joint power control and spectrum access in cognitive radio networks," *Journal of Network and Computer Applications* ,vol. 41, pp. 379-388, May, 2014. Article (CrossRef Link).

[3] Z. Chen, X. Wang and X. Zhang, "Continuous power allocation strategies for sensing-based multiban spectrum sharing," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 11, pp. 2409-2419, November, 2013. Article (CrossRef Link).

[4] R. Zhang and Y. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of Selected Topics In Signal Processing*, vol. 2, no. 1, pp.88-102, February, 2008.Article (CrossRef Link).

[5] Z. Feng, J. Zhang and H. Chen, "Joint beamforming and power allocation for multiple primary users and secondary users in cognitive MIMO systems via game theory," *KSII Transactions on Internet and Information Systems*, vol. 7, no. 6, pp. 1379-1397, June, 2013. Article (CrossRef Link).

[6] J. Liu, G. Kang and Y. Zhu, "Optimal power allocation and outage analysis for cognitive MIMO full duplex relay network based on orthogonal space-time block codes," *KSII Transactions on Internet and Information Systems*, vol. 8, no. 3, pp. 924-944, March, 2014. Article (CrossRef Link).

[7] D. Nguyen, L.-N. Tran, P. Pirinen and M. Latva-aho, "Precoding for Full Duplex Multiuser MIMO Systems:Spectral and Energy Efficiency Maximization," *IEEE Transactions on Signal Processing*, vol. 61, no. 16, pp. 4038-4050, August, 2013. Article (CrossRef Link).

[8] A. Balatsoukas-Stimming, P. Belanovic, K. Alexandris and A. Burg, "On self-interference suppression methods for low-complexity full-duplex MIMO," in *Proc. of 2013 Asilomar Conference on Signals, Systems and Computers*, pp. 992-997, 2013. Article (CrossRef Link).

[9] J. Zhang, O.T. Motlagh, J. Luo and M. Haardt, "Full duplex wireless communication with partia interference cancellation," *2012 Conference Record of the Forty Sixth Asilomar Conference on Signals, Systems and Computers*, pp. 1295-1299, 2012. Article (CrossRef Link).

[10] E. Foroozanfard, O. Franek, A. Tatomirescu, E. Tsakalaki, E. de Carvalho and G.F. Pedersen, "Full-duplex MIMO system based on antenna cancellation technique," *Electronics Letters*, Vol. 50, no. 16, pp. 1116-1117, July, 2014. Article (CrossRef Link).

[11] Y. Liang, Y. Zeng, E.C. Peh and A.T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 7, no.4, pp. 1326-1337, April, 2008. Article (CrossRef Link).

[12] X. Kang, Y. Liang, H.K. Garg and L. Zhang, "Sensing-based spectrum sharing in

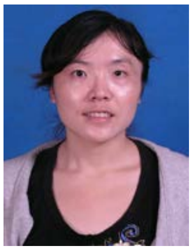
cognitive radio networks," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 8, pp. 4649-4654, October, 2009. Article (CrossRef Link).

[13] G. K. Karagiannidis and A. S. Lioumpas, "An Improved Approximation for the Gaussian Q-Function," *IEEE Communications Letters*, vol. 11, no. 8, pp. 644-646, August, 2007. Article (CrossRef Link).

[14] Q. Li, M. Hong, H. Wai, W. Ma, Y. Liu and Z. Luo, " An alternating optimization algorithm for the MIMO secrecy capacity problem under sum power and per-antenna power constraints," in *Proc. of* 2013 *IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 4359-4363, 2013. Article (CrossRef Link).

[15] J. Jose, N. Prasad, M. Khojastepour and S. Rangarajan, "On robust weighted-sum rate maximization in MIMO interference networks," in *Proc. of 2011 IEEE International Conference on Communications*, pp. 1-6, 2011. Article (CrossRef Link).

[16] S. Huberman and T. Le-Ngoc, "Sequential convex programming for full-duplex single-user MIMO systems," in *Proc. of 2014 IEEE International Conference on Communications*, pp. 5078-5082, 2014. Article (CrossRef Link).



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