

# An Energy Efficient Algorithm Based on Clustering Formulation and Scheduling for Proportional Fairness in Wireless Sensor Networks

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## Abstract

In this paper, we investigate the problem of achieving proportional fairness in hierarchical wireless sensor networks. Combining clustering formulation and scheduling, we maximize total bandwidth utility for proportional fairness while controlling the power consumption to a minimum value. This problem is decomposed into two sub-problems and solved in two stages, which are Clustering Formulation Stage and Scheduling Stage, respectively. The above algorithm, called CSPF\_PC, runs in a network formulation sequence. In the Clustering Formulation Stage, we let the sensor nodes join to the cluster head nodes by adjusting transmit power in a greedy strategy; in the Scheduling Stage, the proportional fairness is achieved by scheduling the time-slot resource. Simulation results verify the superior performance of our algorithm over the compared algorithms on fairness index.

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**Keywords:** Proportional fairness, clustering formulation, scheduling, energy efficiency, power control.

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## 1. Introduction

In wireless sensor networks, research has been carried out to study the fairness of resources allocation, which provide supports in many important scenarios, such as target detection sensor system and wireless multimedia sensor networks. In [1], the authors gave a detailed survey on many kinds of fairness. Among them, proportional fairness is one such criterion, which can bring an allocation balance in a resource scheduling process and prevent the situation where heavy resource users benefit more while the poor ones get little from the resource provider. Hence, it is often used for optimizing scheduling and allocating resources [2, 3].

The existing works that aim at improving network performance mostly focus on maximizing throughput [4], minimizing delay and their trade-off [5]. As one metric of network performance, proportional fairness is well studied in issues [2, 6]. Li Li [6] proposed the basic proportional fairness problem in multi-rate wireless networks and solved it using a distributed and heuristic algorithm [6]. Wei Li investigated the same problem in [6] and proposed a different solution which adopts relaxation to obtain a result that is better than half of the optimal [2]. These two solutions are both solved in multi-rate WLAN via association control.

Another issue encountered in wireless sensor networks is to achieve energy efficiency by adopting power control strategy. With power control, energy consumption can be reduced and the period of lifetime will be extended. Obviously, bigger transmit power results in longer transmission distance, but meanwhile it also leads to shorter network lifetime. However, in previous works [2, 3, 6-9], energy consumption is not considered in system model. Therefore, we hope to seek an energy efficient algorithm for scheduling resources under proportional fairness criteria.

In this paper, we research on a problem about achieving proportional fairness in hierarchical wireless sensor networks, which aims at finding a trade-off between network utilities and energy consumption. From the cross-layer perspective, we formulate the problem as a non-linear mathematical programming and prove that it is an NP-hard problem, which is hard to determine the optimal solution efficiently in a polynomial time. Therefore, we resort to an approximation solution by jointly determining the node association, power control and time allocation. More specifically, we divide the problem into two sub-problems. Consequently, a two-stage algorithm is proposed. In the first stage, node association and transmit power are determined in a greedy strategy. Based on these association coefficients, clusters will be formed by letting sensor nodes join to the designated cluster head nodes. In the second stage, we schedule time resources to achieve fairness. Through the proposed approximation algorithm, the original problem can be solved in a polynomial time. Simulation results indicate that our algorithm has a better performance over compared methods. We also simulate the dynamic adjustment procedure of transmit power allocation.

It is worth noting that, the authors in [10] investigated the similar problem and proposed an algorithm jointly considering power control and AP (Access Point) association. Meanwhile, it should also be pointed out that their transmit power is set equally between all APs and is related to bit rate. In contrast, our power control is devised to have an impact on association while bit rate is set as fixed value. In addition, the procedure of transmit power adjusting is one by one in greedy strategy. Very recently, Sun discussed the problem of combining BS (base station) association and power control in [11]. With fixed BS association, a binary search

strategy for power allocation was used to achieve downlink max-min SINR(Signal to Interference plus Noise Ratio). Indeed, Sun's paper studied a different system model from ours.

The rest of our paper is organized as follows: Related work is in section 2. System model and problem formulation are introduced in section 3. Our proposed algorithm is discussed in section 4. After simulation, the results and discussions are shown in section 5. We conclude this paper in section 6.

## 2. Related Work

Fairness in wireless networks has been studied in the past. Furthermore, to implement fairness is mainly under the domain of data-link layer and resource allocation. In [1], the authors concluded the work of fairness in wireless networks, and they presented a general view of fairness studies. In their opinion, fairness measures can be divided into two groups: Quantitative Fairness Measures and Qualitative Fairness Measures. Specifically, Jain's index and Entropy belong to the quantitative group; Max-min Fairness, Proportional Fairness and the Tian Lan's Model are in the qualitative group. Besides, fairness can be divided into time-based fairness and throughput-based fairness. In issue [12, 13], time-based fairness is introduced. Throughput-based fairness is compared with time-based fairness in issue [13]. As one category of fairness, proportional fairness is researched in [3, 7-9, 14]. The authors in [15] determined reach a network wide proportional fairness in an analytic way. In [7], the authors formulated and studied a generalized proportional fairness problem with user associations to base stations, which a generalized proportional fairness objective function is achieved. In [3], the authors considered how spatial reuse impact on proportional fairness. In [8], the authors studied the proportional fair scheduling (PFS) problem, jointly considering the user selection and utility maximization, in an HM-aided wireless network. For proportional fairness, a fairness criterion is proposed by Kelly [16] and a style of proportional fairness is defined in [2]. Consequently, measuring fairness needs a fairness metric. Specifically, Jain's index [17] is the most widely-used fairness index in issues. Jain's index is between 0 and 1, and a larger value of Jain's index means a better fairness.

Besides fairness, energy efficiency is imminently needed in wireless sensor networks, while the lifetime of nodes is constrained by the energy supplement technology. Moreover, one way to achieve energy efficiency is controlling the transmit power. The problem about energy efficiency always can be an optimization problem, and the objective function can be throughput while under the energy consumption constraint. In [18], the authors formulated an energy-constrained optimization problem for link scheduling, power control and routing in ad hoc wireless networks. The authors in [19] presented a cross-layer design framework in contention-based wireless ad hoc networks for the multiple access problem, and the authors in [4] examined joint link scheduling and power control with throughput improvement. In issue [5], the problem for joint scheduling with either power control or rate control or both is formulated.

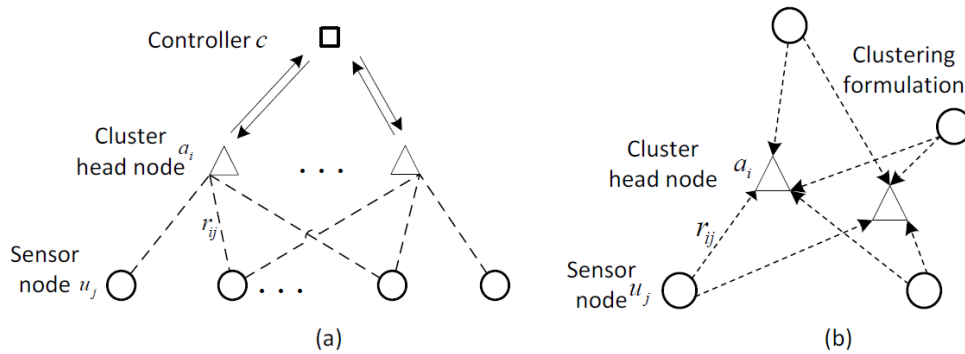
Joint AP association and fairness are proposed by Wei [2]. They proposed their algorithm in static scenario and exposed it to dynamic situation. Similarly, AP association in WLAN can be treated as clustering formulation in wireless sensor networks. Clustering is a classical network formulation method in wireless sensor networks, and the network will be hierarchical. When dealing with clustering, game theory is often investigated [20]. In addition to game theory, interference has an influence on the choice of cluster head nodes, and it further affects the

effective bandwidth allocation. In [3], Douglas et al. demonstrated the problems of applying fairness concepts to wireless networks, which with interference caused by spatial reuse.

### 3. System Model and Problem Formulation

#### 3.1 System Model

We consider a scenario in which wireless sensor networks are hierarchical. In this scenario, there exist  $N$  cluster head nodes named as  $a_1, a_2, \dots, a_i, \dots, a_N$  and  $M$  sensor nodes which can be associated to cluster head nodes and numbered as  $u_1, u_2, \dots, u_j, \dots, u_M$ . Node controller  $c$  runs the proposed algorithm and exchanges messages with cluster head node  $a_i$ . The cluster head node  $a_i$  is a node which can achieve fairness while the allocated resource is time slot in TDMA (Time Division Multiple Access) wireless systems. Sensor node  $u_j$  can only associate with one cluster head node. They are all in the same general area and these nodes can move. We assume that the hierarchical wireless network is dynamic and the network topology varies in case of node mobility. The cluster head nodes have multi-rate ability. Multi-rate wireless sensor network is a kind of wireless sensor network whose links with different data rates coexist.



**Fig. 1.** System scenario. (a) Network model and structure. (b) Nodes in real scenario

**Fig. 1** shows the wireless scenario of system. Sensor node  $u_j$  joins to cluster node  $a_i$  and can only join to one cluster head node.  $u_j$  can choose which  $a_i$  to join in. This procedure of association can be treated as clustering formulation with designated cluster head nodes.

In this paper, we consider a time-slot system with limited bit rate wireless links. The total transmission time of a cluster head node is assumed to be 1. The wireless bit rate of links between  $a_i$  and  $u_j$  is denoted as  $r_{ij}$ , which represents the quality of wireless links. We assume that it is a known parameter. While under SINR interference model,  $I_{ij}$  represents the quality of wireless links and is related to the interference from other wireless links. We have

$$I_{ij} = g\left(\frac{R_{ss_{ij}}}{\sum_{n \in [1, N], m \in [1, M], m \neq j} R_{ss_{nm}} + N_0}\right) \quad (1)$$

where  $I_{ij}$  is the total interference influence from cluster head  $i$  and other nodes to sensor node  $j$ . Specifically,  $n$  and  $i$  are the indexes of cluster head nodes,  $m$  and  $j$  are the indexes of sensor nodes.  $R_{ss_{nm}}$  is the received signal strength from cluster head node  $n$  to sensor node  $m$ .  $\sum_{n \in [1, N]; m \in [1, M], m \neq j} R_{ss_{nm}}$  represents the total interference from all wireless links except himself.  $g$  is an increasing function.  $I_{ij}$  increases with SINR increasing. We assume that the transmit power of cluster head node is higher than that of sensor node. We employ a regular model to represent the wireless channel condition, which is expressed as:

$$R_{ss} = P_t - PL(d_0) - 10\eta \log(dist / d_0) \quad (2)$$

where  $R_{ss}$  represents received signal strength and  $P_t$  is the transmit power. Transmit power minus path loss is received signal strength in reception node. Equation (2) is a large scale path loss model which adopts logarithm distance named as logarithm-distance radio propagation model.  $d_0$  is a reference distance and  $PL(d_0)$  represents the received power in distance  $d_0$ . The  $dist$  represents distance from transmission node to reception node. Symbol  $\eta$  is path loss exponent, which ranges from 2 indoor to 4 outdoor.

The energy consumption is related to the transmit power. In this paper, we consider the power control in cluster head nodes and use vector  $\mathbf{P}_t = [P_{t_1}, P_{t_2}, \dots, P_{t_i}, \dots, P_{t_N}]^T$  to represent the transmit power, where  $P_{t_i}$  represent the transmit power in cluster head node  $a_i$ . We assume that  $P_{t_{\min}} \leq P_{t_i} \leq P_{t_{\max}}$ , where  $P_{t_{\max}}$  is the maximum transmit power and  $P_{t_{\min}}$  is the minimum transmit power in  $a_i$ . We adopt the energy consumption equation in [21]. The total energy consumption can be divided into three parts. We simplify it and assume that transmission nodes have the same length of transmit packet and  $d^n$  varies directly with  $P_{t_i}$ .  $d^n$  represents the  $n$ th power of distance  $d$ , and  $d$  is the distance. We have

$$E_i^c = E^d + \varepsilon^c \cdot P_{t_i} \quad (3)$$

where  $E_i^c$  represents the energy consumption in  $a_i$ .  $E^d$  is a constant parameter related to the transceiver circuit.  $\varepsilon^c$  is assumed to be a constant and is related to the amplifier's electronic parameter.

### 3.2 Problem Formulation

A fairness criterion was proposed by [16] and proportional fairness was described in [2]. We adopt Jain's Fairness Index [17] as the measurement of fairness index, which is denoted as  $J$ . It is an index in the range of [0, 1]. The higher the value, the more equitable. The bandwidth utility for proportional fairness formulation is given by [6].

$$f(x, p) = \sum_{j \in U} \omega_j \log b_j \quad (4)$$

where  $U$  represents the sensor nodes set.  $b_j$  is the effective bandwidth allocated to sensor

nodes from cluster head nodes. Let  $b_j = \sum_{i=1}^N x_{ij} p_{ij} r_{ij}$ ,  $x_{ij}$  is the association between  $u_j$  and  $a_i$ . One node can only associate with one cluster head node, it can be denoted as  $x_{ij} \in \{0, 1\}$ .  $p_{ij}$  is the transmission time that  $a_i$  allocates to  $u_j$ .  $\omega_j$  is the weight of  $u_j$ . We use and reference the optimization formulation in [2, 6] and propose our optimization equations, which are shown as follows:

$$\begin{aligned}
 & \text{maximize} \quad \sum_{j \in U} \omega_j \log \left( \sum_{i=1}^N x_{ij} p_{ij} r_{ij} \right) & (a) \\
 & \text{minimize} \quad \sum_{i=1}^N (E^d + \varepsilon^c \cdot P t_i) & (b) \\
 & \text{s.t.} \quad \sum_{i=1}^N x_{ij} = 1 & (c) \\
 & \quad \quad x_{ij} \in \{0, 1\} & (d) \\
 & \quad \quad \sum_{j=1}^M x_{ij} p_{ij} = 1 & (e) \\
 & \quad \quad p_{ij} \in [0, 1] & (f) \\
 & \quad \quad i \in [1, N], j \in [1, M] & (g)
 \end{aligned} \tag{5}$$

This group of formulations is an NP-hard problem. We prove it in **Proposition 1**. We name it as AFME Problem (Achieving Fairness and Minimizing Energy Consumption Problem). The unknown parameters that need to figure out are  $x_{ij}$ ,  $p_{ij}$  and  $P t_i$ . We aim at achieving proportional fairness for bandwidth allocation while ensuring a less energy consumption. Objective function (a) is to maximize bandwidth utility for proportional fairness. Objective function (b) represents minimizing the sum of energy consumption in each cluster heads. Constraint (c) and (d) say that node  $u_j$  can only associate to one cluster head node  $a_i$  simultaneously. Constraint (e) says that the total transmission time of  $a_i$  is 1. Constraint (f) and (g) define the range of  $p_{ij}$ ,  $i$  and  $j$ .

**Proposition 1.** Formulation (5) is an NP-hard problem.

**Proof.** We bring in a problem named GPF1 (Generalized Proportional Fairness) introduced in paper [7]. This problem has been proved to be NP-hard. As same as GPF1, GPF2 (restricted version of the GPF1 problem) is also an NP-hard problem. What we need to do is to find a polynomial-time reduction function  $F$  which makes  $GPF2 \leq_p MFME$ . Symbol  $\leq_p$  represents polynomial-time reduction. Since GPF2 is an NP-hard problem, we can judge that AFME problem is also an NP-hard problem if there have a  $F$ . GPF2 problem can be shown as follows [7]:

$$\begin{aligned}
 & \text{maximize} \quad \sum_{u \in U} \sum_{a \in S_u} x_{ua} \log \left( r_{ua} \frac{G(y_a)}{y_a} \right) & (a) \\
 & \text{s.t.} \quad \sum_{a \in S_u} x_{ua} = 1, \forall u \in U & (b) \\
 & \quad \quad y_a = \sum_{u: a \in S_u} x_{ua}, \forall a \in A & (c) \\
 & \quad \quad x_{ua} \in \{0, 1\} & (d)
 \end{aligned} \tag{6}$$

where a user  $u$ 's average data rate associating with a BS(Base Station)  $a$  is denoted  $r_{ua}$ .  $x_{ua}$  is the association variable,  $x_{ua}=1$  if  $u$  is associated with  $a$ .  $S_u = \{a \mid r_{ua} > 0, \forall a \in A\}$ ,  $A$  is the set of BSs. In addition,  $y_a = \{u \mid x_{ua} = 1, \forall u \in U\}$ ,  $U$  is the set of users.  $G(y_a) = E\{\max_{u: x_{ua}=1} Y_{ua}\}$ , where  $Y_{ua}$  are independent and identically distributed copies. Besides,  $\gamma_{ij} = G(y_a)r_{ua} / y_a$  where  $\gamma$  represents the actual bandwidth allocation to user  $u$  by the network.

First, we can find out that AFME and GPF2 have the same meaning of constraints. They have  $\sum_{i=1}^N x_{ij} = 1, x_{ij} \in \{0,1\}$ .  $\sum_{j=1}^M x_{ij} p_{ij} = 1$  and  $y_a = \sum_{u: a \in S_u} x_{ua}$  have the same meaning and the total allocation time in cluster head nodes is assumed to be 1 in problem AFME. Second, AFME has two object functions while GPF2 has one objective function. Here, we use the method of weighted sum of objectives as a single objective to deal with this optimization problem, which will formulate a new utility function. It can be expressed as:

$$\lambda_1 \left( \sum_{j \in U} \omega_j \log \left( \sum_{i=1}^N x_{ij} p_{ij} r_{ij} \right) \right) - \lambda_2 \sum_{i=1}^N E_i^c \quad (7)$$

where  $\lambda_1$  and  $\lambda_2$  are the weight of proportional fairness function and energy consumption function. Besides, both of them can also be normalized coefficients. Furthermore, parameter  $\lambda_1$  and  $\lambda_2$  can be set utilizing expert's experience. Consequently, we maximize the total utility function with respect to bandwidth allocation for proportional fairness while controlling the power consumption to a minimum value, which will bring an energy efficient bandwidth assignment for proportional fairness. Finally, since the two object functions of two problems are polynomial, we can find a polynomial time function  $F$  that makes  $GPF2 \leq_p MFME$ . Then the AFME is an NP-hard problem.

## 4. Proposed Algorithm

According to the optimal model, there will be a balance between fairness and energy consumption. The unknown parameters which need to be solved are the association relationship  $x_{ij}$ , scheduling time slot result  $p_{ij}$  and power allocation information  $Pt_i$ . To solve AFME problem, we divide the problem into two sub-problems and each sub-problem corresponds to one stage. The first stage formulates the network structure, and sensor nodes join to the cluster head nodes. The first sub-problem is in the stage of clustering formulation based on SINR interference model and power control, which can obtain  $x_{ij}$  and  $Pt_i$ ; in the second stage, after the networks structure has formulated, we use the information got from the first stage, and then scheduling. We can obtain  $p_{ij}$  in this stage. The resource in this stage that can be allocated is time-slot resource.

### 4.1 Clustering Formulation Stage

In this stage, we firstly focus on figuring out the  $x_{ij}$ , which means that sensor node  $u_j$  joins to cluster head node  $a_i$ . This process can be treated as clustering formulation while cluster head nodes are designated. It deletes the unimportant potential association depending on  $r_{ij}$ ,  $Rss$

and  $E_i^c$ . We set up a liner function to express the contributions of each factors to the final association result.

$$\Delta_{ij} = \zeta_1 \eta_{ij} + \zeta_2 I_{ij} - \zeta_3 E_i^c \quad (8)$$

where  $\Delta_{ij}$  represents the association strength with respect to bit rate, interference and transmit power. For each  $u_j$ , we choose the maximum  $\Delta_{ij}$  and find the corresponding  $a_i$ . Let  $u_j$  associate to  $a_i$  and set  $x_{ij} = 1$ , otherwise set as 0.  $\zeta_1, \zeta_2$  and  $\zeta_3$  are normalizing ratio, and it was set considering the impact of weight. In the first part  $\eta_{ij} = r_{ij} / (\sum_{j=1}^M r_{ij})$ , taking the bit rate's influence into account. The second part introduces the impact of interference, and the third part is the impact of energy consumption. It can be rewritten in the following format:

$$\Delta_{ij} = \zeta_1 r_{ij} / (\sum_{j=1}^M r_{ij}) + \zeta_2 g(\text{SINR}(\frac{Rss_{ij}}{\sum_{n \in [1, N], m \in [1, M], m \neq j} Rss_{nm} + N_0})) - \zeta_3 (E^d + \varepsilon^c \cdot Pt_i) \quad (9)$$

In this formulation, we first find the unknown parameters and can figure out the relationship between input parameters and output result.  $I_{ij}$  is related to  $Pt = [Pt_1, Pt_2, \dots, Pt_i, \dots, Pt_N]^T$ . We need to find a new way to figure out the maximum  $\Delta_{ij}$ .

We adopt a greedy strategy to deal with the  $Pt$  in the process of computing  $\Delta_{ij}$ . Since  $Pt_i \in [Pt_{\min}, Pt_{\max}]$ , we discrete  $Pt_i$  into  $K$  pieces, and  $k$  is the index of pieces. Thus the value matrix can be shown as follows:

$$Pt = [Pt_1, Pt_2, \dots, Pt_i, \dots, Pt_N]^T \longrightarrow \begin{bmatrix} Pt_{11} & Pt_{21} & \dots & Pt_{N1} \\ Pt_{12} & Pt_{22} & \dots & Pt_{N2} \\ \dots & \dots & Pt_{ik} & \dots \\ Pt_{1K} & Pt_{2K} & \dots & Pt_{NK} \end{bmatrix}^T \quad (10)$$

$$Pt_{ik} = Pt_{\min} + (k-1) * (Pt_{\max} - Pt_{\min}) / (K-1) \quad (11)$$

where  $Pt_{ik}$  represents the  $k$ th piece of  $Pt_i$ . To find the maximum  $\Delta_{ij}$  for each  $j$ , different  $Pt_{ik}$  has different  $\Delta_{ij}$ . For different  $a_i$ , we select a  $Pt_i$ , and we have  $(Pt_{ik}, r_{ij}) \rightarrow \Delta_{ijk}$ , for each  $a_i$ , we choose  $Pt_{ik}$  and the related  $k$  in the guidance equation which can be expressed as:

$$\max_k \sum_{j=1}^M \Delta_{ijk} \quad (12)$$

For different  $a_i$ , we adopt a greedy strategy to avoid uncertain situation. For  $a_i$ , we need to know about  $Pt_{1k_1}, \dots, Pt_{(i-1)k_{i-1}}, Pt_{ik}, Pt_{(i+1)k_{i+1}}, \dots, Pt_{Nk_N}$  according to  $\Delta_{ij}$  when selecting  $Pt_{ik}$ , where  $k_1 \dots k_{i-1} k_{i+1} \dots k_N$  is the chosen  $k$  for each  $a_i$ . We vary  $Pt_{ik}$  for each  $k$  and fix the value of  $Pt_{1k_1}, \dots, Pt_{(i-1)k_{i-1}}, Pt_{ik}, Pt_{(i+1)k_{i+1}}, \dots, Pt_{Nk_N}$ . We found  $k$  according to formulation (12). We update the value of  $Pt_{ik}$  at each round for  $a_i$ . Finally, we can find appropriate  $k$  for each column in the matrix, then  $Pt_{ik}$  can be figured out. According to formulation (12) with the



corresponding  $i$  and  $k$ , nodes association  $x_{ij}$  can be determined.

In the procedure of adjustment, the transmit power varies from the minimum value and find  $k$  to satisfy the strategy. It can bring a less energy consumption and make the algorithm energy efficient.

## 4.2 Scheduling Stage

After the clustering formulation stage, we have figured out  $x_{ij}$  and  $P_{t_{ik}}$ , while the network structure formulated. In this stage, only  $p_{ij}$  is left to be determined. Different  $p_{ij}$  results in a different fairness performance. The scheduling is needed in this stage. We assume that the transmit power are fixed since the dynamic power allocation adjustment has finished. This optimal problem can be simplified as follows:

$$\begin{aligned} & \text{maximize} \quad \sum_{j \in U} \omega_j \log \left( \sum_{i=1}^N x_{ij} p_{ij} r_{ij} \right) \\ & \text{s.t.} \quad \begin{cases} p_{ij} = 0 & \text{if } x_{ij} = 0 \\ \sum_{j=1}^M x_{ij} p_{ij} = 1 & \text{if } x_{ij} = 1 \end{cases} \\ & \quad p_{ij} \in [0, 1] \end{aligned} \quad (13)$$

It is solve\_opt (Algorithm 1) in the proposed algorithm. As is shown in the equation,  $p_{ij}$  is corresponding to  $x_{ij}$ . When  $x_{ij}=0$ , it means that  $u_j$  has no association with  $a_i$ , not to mention that  $a_i$  allocates time-slot to  $u_j$ . Otherwise, the sum of time-slot that  $a_i$  allocates to  $u_j$  is 1 and this has been assumed. This model can be solved in polynomial time and has an optimal solution.

**Proposition 2.** Formulation (13) can be solved in polynomial time. It satisfies the KKT necessary and sufficient condition.

**Proof.** We make matrix  $p_{ij}$  head-to-tail transforms to be a vector  $\mathbf{P}_v$  by column, and  $\sum_{i=1}^N x_{ij} p_{ij} r_{ij}$  can be expressed as  $\mathbf{\Omega} \mathbf{P}_v$ , where  $\mathbf{\Omega}$  represents a known constant matrix. Formulation (13) can be  $\mathbf{\Gamma} = \mathbf{\Phi} \log(\mathbf{\Omega} \mathbf{P}_v)$ , where  $\mathbf{\Gamma}$  represents the target function and  $\mathbf{\Phi}$  represent the known constant matrixes corresponding to  $\mathbf{\omega}$ . Equal constraint can be expressed as  $\mathbf{Y} \mathbf{P}_v - \mathbf{1} = 0$ , where  $\mathbf{Y}$  is a constant matrix corresponding to  $\mathbf{x}$ . Then KKT necessary and sufficient condition can be expressed as

$$\begin{aligned} & \mathbf{\Gamma}' - \nu (\mathbf{Y} \mathbf{P}_v - \mathbf{1})' = 0 \\ & (\mathbf{\Phi} \log \mathbf{\Omega} + \mathbf{\Phi} \log \mathbf{P}_v)' - \nu \mathbf{Y} = 0 \\ & \mathbf{\Phi} / \mathbf{P}_v - \nu \mathbf{Y} = 0 \end{aligned} \quad (14)$$

where  $\nu$  represents a non-negative vector.  $\bar{\mathbf{P}}_v = \mathbf{\Phi} / \nu \mathbf{Y}$  and  $\bar{\mathbf{P}}_v$  is the KKT point. Due to the fact that logarithm function is a convex function and equal constraint is linear, KKT point  $\bar{\mathbf{P}}_v$  is the optimal result for this non-liner problem. Formulation (13) can be solved in polynomial time. It satisfies the KKT necessary and sufficient condition.

### 4.3 CSPF\_PC Algorithm

Based on the stage mentioned above, we proposed an algorithm which is divided into two stages. In each stage, we figure out part of the optimal problem. The first stage figures out the clustering formulation problem with respect to power control and finds out the association information and proper transmit power in cluster head nodes. The second stage figures out the time-slot allocation or scheduling problem. The target function for this optimal problem is to achieve proportional fairness in cluster head nodes while considering minimizing the energy consumption. This algorithm can be shown in [Algorithm 1](#).

[Algorithm 1](#) indicates the key steps of proposed method to figure out this kind of problem. Row 1 says that system can obtain the channel state information by exchanging messages. Row 2~9 indicates the key steps of Clustering Formulation Stage, which has been introduced before. Row 10 is the step of Scheduling Stage, and the problem can be solved in classical algorithms through mathematical tools. Row 11 indicates the algorithm's result.

Algorithm 1	Clustering formulation and scheduling for proportional fairness based on power control algorithm( CSPF_PC Algorithm)
Input	1、 Bit rate $r_{ij}$ between $a_i$ and $u_j$ 2、 Weight $\omega_j$ for each $u_j$ . 3、 The range of transmit power $[P_{t_{\min}}, P_{t_{\max}}]$
1	Cluster head nodes send packets and nodes return response packets including channel stage information
2	For each $a_i, i \in [1, N]$ $u_j, j \in [1, M]$ do
3	$\eta_{ij} = r_{ij} / (\sum_{j=1}^M r_{ij})$
4	$I_{ij} = g(SINR(Rss_{ij} / \sum_{n \in [1, N]; m \in [1, M], m \neq j} Rss_{nm} + N_0))$
5	$E_i^c = E^d + \varepsilon^c \cdot P_{t_i}$
6	$\Delta_{ij} = \zeta_1 \eta_{ij} + \zeta_2 I_{ij} - \zeta_3 E_i^c$
7	Choose $P_{t_{ik}}$ in the described greedy strategy $\max_k \sum_{j=1}^M \Delta_{ijk}$ for each $a_i$
8	set $x_{ij} = 1$ if $\Delta_{ij} == \max(\Delta_{ij})$ for each $u_j$ , otherwise $x_{ij} = 0$
9	end
10	$p_{ij} = \text{solve\_opt}(x_{ij}, r_{ij}, \omega_j)$
11	Controller $c$ sends results to each cluster head node $a_i$

The algorithm is finite and can have an optimal result. In the first stage, for  $a_i$  and  $u_j$ , the loop will end while  $i \in [1, N]$  and  $j \in [1, M]$ . When choosing  $P_{t_{ik}}$  and finding the  $\max(\Delta_{ij})$ , the procedure is finite, for the discrete number is limited and  $j \in [1, M]$ . In the second stage, there will be a KKT point in the optimal problem and it can be solved using a classical algorithm.

In the first stage, the computation complexity of loops is  $\Theta(MN)$ , and choosing  $P_{t_{ik}}$  can cost  $\Theta(NK)$  for adopting greedy strategy. It can be treated as a branch bounding method which selects a branch that has the suboptimal value of transmit power, cutting down the branches at

each time for reducing the computation complexity evidently. The computation complexity of choosing maximum  $\Delta_{ij}$  is included in the loop of  $j$ . In the second stage, since it can be solved in polynomial time, we treat it as  $\Theta(MN)$ . Finally, the computation complexity of this problem is  $2*\Theta(MN)+\Theta(NK)$ .

## 5. Simulation and Discussion

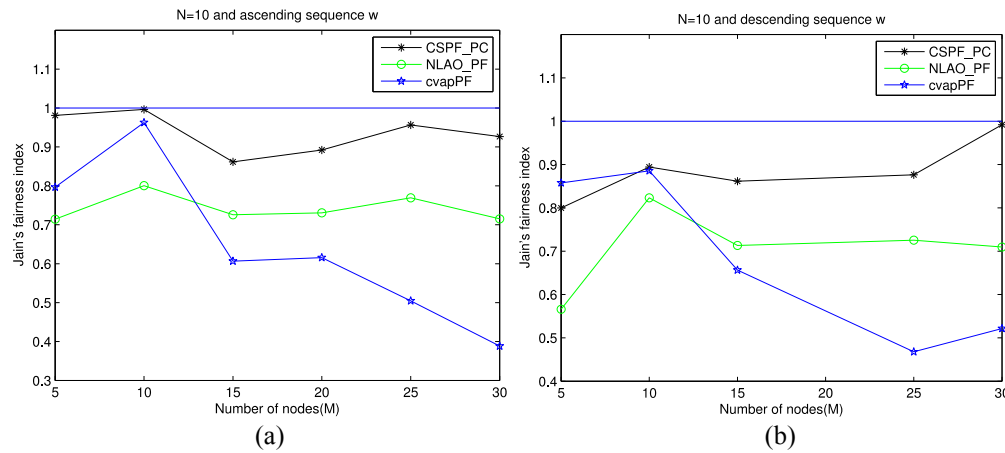
We simulate the performance of our new proposed algorithm compared to NLAO-PF algorithm [2] and cvapPF algorithm [6]. We choose these two algorithms for the reason that this paper comes from the two compared issues, and the flows of algorithms are similar. The AP association can be treated as clustering formulation in wireless sensor networks.

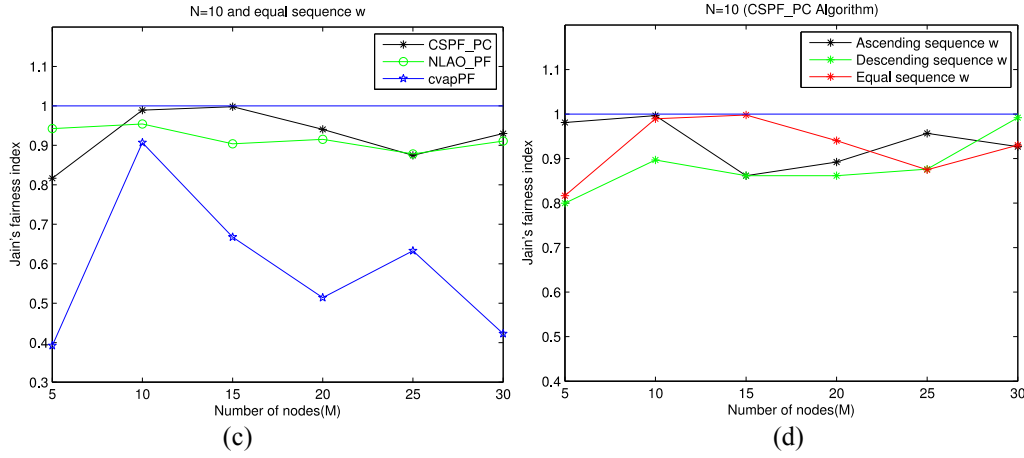
We adopt Jain's fairness index as the compared metric in this paper. Jain's fairness index is set between 0 and 1 and widely used in issues about fairness. The larger Jain's fairness index, the better fairness. The definition of Jain's fairness index in this paper is

$$J = \frac{[\sum_{j=1}^M b_j]^2}{M \sum_{j=1}^M b_j^2} \quad (15)$$

where  $J$  represents Jain's fairness index and  $b_j$  is bandwidth.

We set some of the parameters as follows: reference distance  $d_0=1[m]$ , path loss for reference distance  $d_0$  as  $PL(d_0)=55[dB]$ , noise  $N_0=-80[dB]$ , path loss exponent  $\eta=4$ , the width of simulation square  $L=200[m]$ . We set maximum value of transmit power  $P_{t_{\max}}=40[mW]$  and minimum value of transmit power  $P_{t_{\min}}=10[mW]$ ,  $K=10$ . The area of simulation network is set to be  $200m \times 200m$ . In our simulation scenario, nodes are randomly deployed in a square region and  $L$  is the width of square.





**Fig. 2.** Simulation results of different  $M$  and  $\omega_i$ . (a) Ascending sequence  $\omega_i$ ; (b) descending sequence  $\omega_i$ ; (c) equal sequence  $\omega_i$ ; (d) different sequence  $\omega_i$  for CSPF\_PC algorithm.

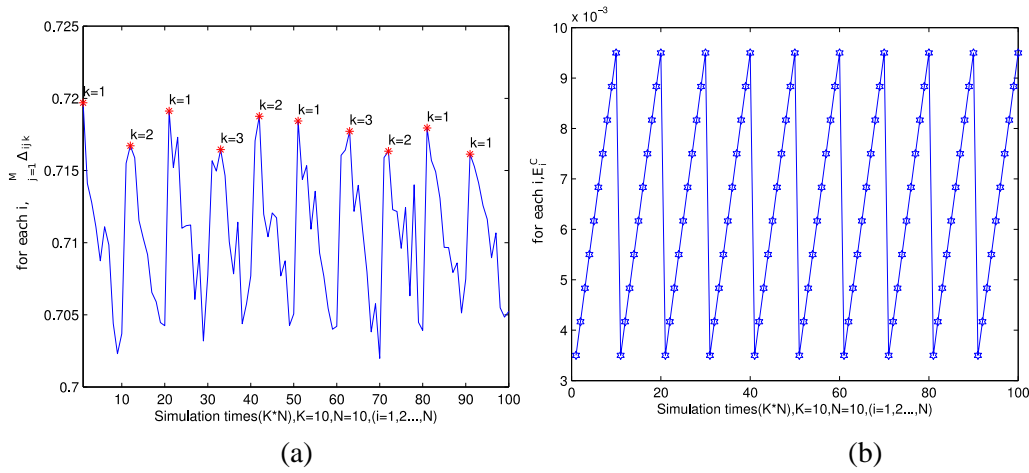
We compare the algorithms under different  $M$  values. We use  $N = 10$  and let  $M$  varies in the range of  $[5, 30]$ .

**Fig. 2** shows the impact of nodes size  $M$  and weight  $\omega_i$  on fairness index. We firstly set weight  $\omega_i$  in an ascending sequence, and we obtain the Jain's fairness index of three algorithms as is shown in

**Fig. 2(a).** As the number of nodes increases, the fairness index remains above 85% and CSPF\_PC increases the fairness index by 18%~37% compared with NLAO\_PF algorithm. Then we evaluate the descending sequence of  $\omega_i$  and equal sequence of  $\omega_i$ , and we can observe that when  $N=M$ , the fairness index of three algorithms will get its locally optimal point. This is due to the fact that  $\omega_i$  is in the fairness formulation, which has a great impact on final result.

**Fig. 2(b)** and (c) show that as  $\omega_i$  becomes equal, the fairness index becomes stable except cvapPF algorithm. From these three figures, we can observe that CSPF\_PC has a better performance on fairness index than the other two algorithms as  $M$  and  $\omega_i$  vary. We use different values of  $\omega_i$  for comparison. First we get a sequence of  $\omega_i$  which satisfies  $\sum_{j=1}^M \omega_j = 1$  and order it in an ascending sequence. Then we get descending and equal sequence.

**Fig. 2(d)** indicates that the impact of different weight  $\omega_i$  on fairness index using CSPF\_PC algorithm. We average the results and it is 0.9358, 0.8814 and 0.9248. It shows that ascending sequence  $\omega_i$  works best in this situation.



**Fig. 3.** Simulation result of transmit power adjustment procedure using CSPF\_PC algorithm. (a) The procedure of  $\Delta$  varies using greedy strategy. (b) Transmit power adjustment procedure.

**Fig. 3(a)** shows the procedure of  $\sum_{j=1}^M \Delta_{ijk}$  varies when adopting fixed  $i$  for each iteration. We choose the  $k$  using greedy strategy labeled in the figure and use it in  $Pt_{ik}$ . Then, we can find out the suitable value in this procedure. **Fig. 3(b)** shows the procedure of transmit power varies. From **Fig. 3**, we can see that  $\sum_{j=1}^M \Delta_{ijk}$  for each  $i$  varies while transmit power varies, but the variation trend is not synchronous. The reason due to the fact that the weight  $\zeta_1, \zeta_2, \zeta_3$  have different impact on the result in this procedure.

## 6. Conclusion

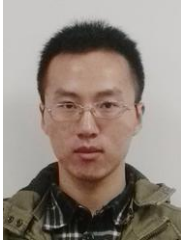
In this paper, we developed a polynomial-time algorithm which is called CSPF\_PC algorithm to solve the problem of achieving proportional fairness in wireless sensor networks. The goal is to maximize bandwidth utility with less energy consumption under the constraints that (i) sensor node can only associate to one cluster head node, and (ii) the total transmission time is assumed to be 1. Through the algorithm, optimization variables were determined in two stages. Simulation results demonstrate that CSPF\_PC outperforms in terms of fairness metric in different input parameters.

In future research, we will consider to find an optimal solution of to the problem. In addition, we will also consider power allocation in both of the stages. Moreover, we will consider a contention based scenario and develop a distributed algorithm in future.

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