An amplify-and-forward relaying scheme based on network coding for Deep space communication

Wangmei Guo[1], Junhua Zhang[2], Guiguo Feng[1], Kaijian Zhu[2] and Jixiang Zhang[1]
[1] State Key Laboratory of Integrated Services Networks (ISN), Xidian University
Xi'an, 710071, China
[e-mail: wangmeiguo@xidian.edu.cn, 365110955@qq.com, 1083221267@qq.com]
[2] Xi'an Satellite Control Center
Xi'an, 710043, China
[e-mail: zjhcn_2000@sohu.com, goodzkj7612@126.com]
*Corresponding author: Wangmei Guo

Received September 21, 2015; revised November 21, 2015; accepted December 10, 2015; published February 29, 2016

Abstract

Network coding, as a new technique to improve the throughput, is studied combined with multi-relay model in this paper to address the challenges of long distance and power limit in deep space communication. First, an amplify-and-forward relaying approach based on analog network coding (AFNC) is proposed in multi-relay network to improve the capacity for deep space communication system, where multiple relays are introduced to overcome the long distance link loss. The design of amplification coefficients is mathematically formulated as the optimization problem of maximizing SNR under sum-power constraint over relays. Then for a dual-hop relay network with a single source, the optimal amplification coefficients are derived when the multiple relays introduce non-coherent noise. Through theoretic analysis and simulation, it is shown that our approach can achieve the maximum transmission rate and perform better over single link transmission for deep space communication.

Keywords: Deep space communication, amplify-and-forward, multi-relay, analog network coding

This research was supported by the National Natural Science Foundation of China (Grant Nos. 61271174 and No. 61301178). We express our thanks to the experts in Xidian University and Xi'an Satellite Control Center for their beneficial suggestions.

http://dx.doi.org/10.3837/tiis.2016.02.013 **ISSN : 1976-7277**

1. Introduction

Deep space communication plays a crucial role in particular for operations, efficient science-data downlink, and public outreach in case of human missions [1-2]. There are some specific challenges that would not encounter in the wireless communications on the earth, such as the long distance transmission resulting into great loss combined with the limited power on the space craft [3-6]. To solve the problems, lots of ways are considered in deep space communication, such as image compression and channel coding as well as application layer coding mechanism are simultaneously applied for image transmission over deep space channels[7,8]. At present, single link is always adopted to transmit information between the space craft and the earth, however, it is not an efficient way for the case of the long distance transmission and power limit [1]. Multi-relay structure or space network is a trend for future exploration demand [4,5].

In the future deep space exploration, the capacity of the communication system will have a huge demand [5]. Network coding is a new technique which allows intermedia nodes make a combination of its received messages and then send out, rather than just store and forward in the traditional communication network [9,10]. It is proved that there are advantages in throughput, load equalization, power deduction and so on [9-12]. Therefore, network coding is introduced to address the problems in deep space communication, which maybe bring with valuable things. Fountain coding is a application of network coding to overcome communication interruption [13]. A self-cooperative network coding is proposed for reliable transmission of the large file in deep space and low transmission delay, where sender makes use of time diversity technologies to offer cooperative information by itself [14].

Multi-relay structure is considered in this paper. On the surface, the increased terminals will introduce more overhead, more noise and lower reliability. In fact, it can utilize a small amount of space crafts to implement link segmentation to deduce the link loss at the same time. It will be a high benefit-cost ratio method when the related technique is introduced to guarantee the reliability and low noise. However, how to design the scheme for the relay nodes to cooperate with each other to maximize the transmission rate is a problem worth to study.

There are three main relaying schemes currently, including Amplify-and-Forward (AF), Decode-and-Forward (DF) and Compress-and-Forward (DF)[11,15]. The last two schemes have higher complexity while AF is only with linear complexity, which make it easy to carry out in deep space communication. Combined with analog network coding, an efficient amplify-and-forward (AFNC) scheme for deep space multi-relay network communication is proposed, where non-source nodes allow to combine its received messages according to the coding coefficients chosen random or fixed before transmission. Meanwhile, the sum power constraint over relays is set to lower the energy consumption inspired by the study of power allocation in [16]. A single source and dual-hop multi-relay network is considered in this paper. The amplification coefficient is designed to maximize the received signal-to-noise ratio (SNR) at sink node when the multiple relays introduce non-coherent noise and are subject to sum-power constraint. Then, the optimal amplification coefficient is derived in details for a dual-hop two-relay network with a single source. It is shown through theoretic analysis and simulation that this method is an efficient way to improve the channel capacity.

The organization of this paper is as follows. First, the characters of deep space communication and the principle of network coding are introduced as preliminaries in Section 2. Our model and the AFNC scheme are presented in Section 3. Then an illustrative example and the simulation are given in Section 4 and 5, respectively, which show that our approach have advantages in the transmission rate.

2. Definitions and Preliminaries

2.1 Deep Space Communication

In deep space communication, let P_T be the transmit power at source and P_r be the received power at sink node, then we have

$$P_r = P_T \cdot G_{Tx} \cdot G_{Rx} \cdot \frac{\lambda^2}{(4\pi d)^2} \tag{2-1}$$

where G_{Tx} is the gain at the transmit antenna, G_{Rx} is the gain at the receiving antenna, d is the transmission distance, and λ is the wavelength of the signal. The coefficient $G_{Tx} \cdot G_{Rx} \cdot \frac{\lambda^2}{(4\pi d)^2}$ indicates the power loss at the link.

Corresponds to the signal, the loss can be presented as

$$H_c = \sqrt{G_{Tx} \cdot G_{Rx} \cdot \frac{\lambda^2}{(4\pi d)^2}} = \sqrt{G_{Tx} \cdot G_{Rx}} \cdot \frac{\lambda}{4\pi d} = \frac{K_{sh}}{d} \tag{2-2}$$

where $\sqrt{G_{Tx} \cdot G_{Rx}} \cdot \frac{\lambda}{4\pi} = K_{sh}$.

That is, the transmission between the space crafts can be formulated as

$$y = H_c x + z \tag{2-3}$$

where x is the transmit signal, and z is the noise for deep space communication links which can be normalized as a $N(0,1)$ Gaussian noise in general [17,18].

2.2 Network Coding

Network coding is a fusion of routing and coding. Its main idea is that the non-source nodes can make a linear or nonlinear combination of its received messages and then send out, where the intermedia nodes play a role of encoder or signal processer. It is proved that this technique can improve the throughput[9]. Analog network coding is a method used in physical network coding, which is mainly based on the relay amplify and forward mode[11,15]. Analog network coding can directly process the modulated signal, and its focus is how to recover a lossy signal rather than process the signal by mathematics.

Communication is always modeled as a directed graph $G(V,E)$, where V is the set of nodes, and E is the set of links. For a non-source node $j \in V$, its coding process is defined as

$$x_j = \sum_{i \in In(j)} \alpha_{i,j} y_i \tag{2-4}$$

where $In(j)$ denotes the set of incoming adjacent nodes of node j, and y_i is the signal received at node j from node i, $\alpha_{i,j}$ is the coding coefficients, x_j is the transmit signal at node j after the coding process. At the sink node, the corresponding inverse transformation is processed to the received mixed messages to recover the source messages.

3. Network Model and AFNC Scheme

3.1 Network Model and Problem Formulation

Consider the deep space communication between the planets and the Earth in **Fig. 1**. Direct link (red link) is used at present, and it is very high cost to overcome the huge path loss and delay when the messages are transmitted directly between them. It is proved that relay network model could reduce the path loss and obtain better performance. On the other hand, a small amount of space crafts that have been sent into deep space could be used to build multiple paths to increase the channel capacity. A relay satellite divide the distance from Mars to Earth into parts, which reduce the total transmission power to some extent.

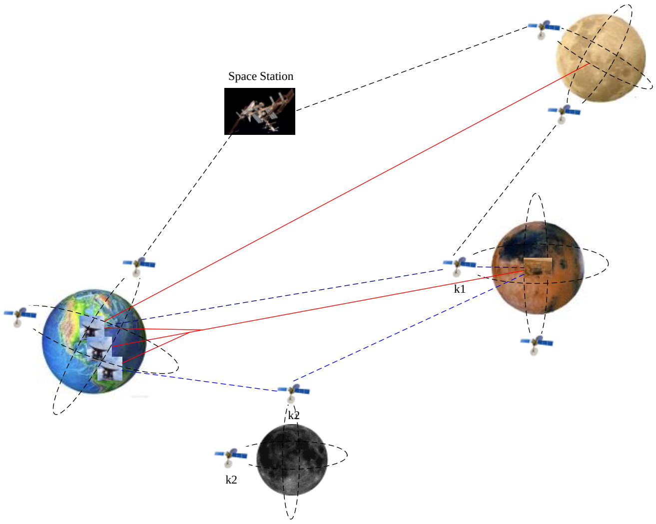

Fig. 1. Deep space relay network

A multi-relay two-hop model is considered in this paper, as in **Fig. 2**. The set of the relays is denoted as $\mathrm{K} = \{1, 2, ..., K\}$, and the distance d_{ij} between node i and node j is labeled as in the figure. Half duplex and AF scheme is considered over relays. Combined with network coding, we propose AFNC relaying scheme, that is, non-source nodes can make a combination of its received signals, and the chosen rule of the coefficients is to maximize SNR to achieve the maximum transmission rate.

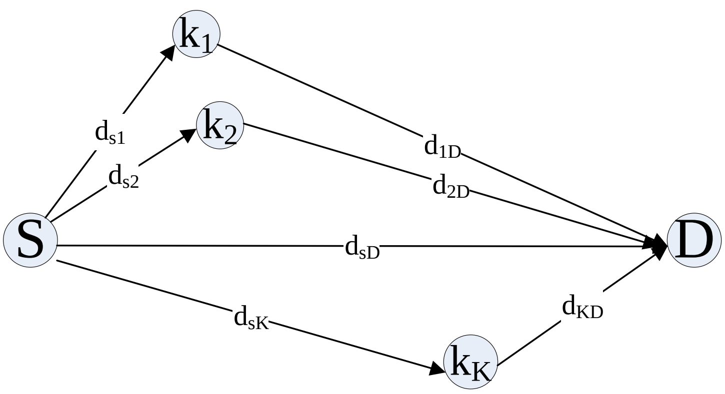

Fig. 2. Deep space relay network

Assume that each node k (non-source) simultaneously receives the signal from the source node S via different relay path, and there is no interference for all the output of channels. For this reason, we can ignore the time index, and the signal received at relay k can be expressed as

$$y_k = h_{s,k} x_s + z_k \tag{3-1}$$

where $h_{s,k} = \dfrac{K_{sh}}{d_{sk}}$ is the channel response which represents the power loss in terms of distance, and z_k is independent Gaussian noise with mean 0 and variance δ_k^2. For simple, the gaussian noise at all relays is denoted by $\mathbf{z} = [z_1, z_2, ..., z_K]^T \in C^{K\times 1}$, and its correlation matrix is $\mathbf{K_R} \succ \mathbf{0} \in C^{K\times K}$ with δ_k^2 as the kth diagonal element. For a given antenna gain and communication setup, K_{sh} is considered as a constant in this paper. Hence, the received signal power at relay k is

$$\mathrm{P}_{R,k} = E[|y_k|^2] = |h_{s,k}|^2 P_s + \delta_k^2, \quad \forall k \in \mathrm{K} \tag{3-2}$$

Then, the relays amplify the received signal according to the coding coefficients, i.e., the transmission signal $x_k \in C$ at relay k can be expressed as

$$x_k = \alpha_k y_k \tag{3-3}$$

where $\alpha_k \in C$ is the amplify coding coefficeint at the relay k. So we have the transmission signal power at relay k as

$$P_{T,k} = E[|x_k|^2] = |\alpha_k|^2 (|h_{s,k}|^2 P_s + \delta_k^2), \quad \forall k \in \mathrm{K} \tag{3-4}$$

In order that the increased relays will not increase too much power consumption, we set a sum-power constraint over relays by the following formula

$$\sum E\left[|x_k|^2\right] = \sum P_{T,k} \le P_{sum\_\max} \tag{3-5}$$

Compare with the transmission delay, the process time could be ignored. So we assume all relay nodes are instantaneous. Therefore, the received signal at the sink node is

$$y_D = \sum_{j\in In(k)} h_{k,D} \alpha_k (h_{s,k} x_s + z_k) + h_{s,D} x_s + z_D \tag{3-6}$$

where $h_{k,D} = \dfrac{K_{sh}}{d_{kD}}$ is the channel response from relay k to sink D, and $h_{s,D} = K_{sh} / d_{sD}$ is the channel response directly from source node to sink node. For simple, we denote all $h_{s,k}$ as a $K\times 1$ column vector $\mathbf{h}_s = [h_{s,1}, h_{s,2}, ..., h_{s,K}]^T$, and denote all $h_{k,D}$ as a $1\times K$ row vector $\mathbf{h}_D = [h_{1,D}, h_{2,D}, ..., h_{K,D}]$. A coding coefficeint matrix is formed as $A = diag(a_1, a_2, ..., a_K) \in C^{K\times K}$. Therefore, the signal received at sink can be written in matrix form as

$$y_D = \mathbf{h_D} \mathbf{A} (\mathbf{h_s} x_s + \mathbf{z}) + h_{s,D} x_s + z_D \tag{3-7}$$

A coding sequence with length n and $\left[2^{nR}\right]$ codewords is used in our approach, the error probability of which approaches to 0 as the code length $n \to \infty$. The codebook at source node is Gaussian codebook with distrubition $N\left[0, P_s\right]$, where P_s is transmit power at source. The relay satellites amplify and forward the received messages without delay, and the amplification coefficients α_k are chosen such that the sum-power constraint (3-5) holds.

We will take the SNR as the measure of the quality. So the aim of the amplify coding coefficeint in our AFNC scheme is to maximize the SNR under the sum power constraint, which is formulated as follows

$$\begin{aligned} &\max_{\alpha} \quad \frac{|\boldsymbol{\alpha}^H \mathbf{H}_D \mathbf{h}_s + h_{s,D}|^2 P_s}{\boldsymbol{\alpha}^H \mathbf{H}_D \mathbf{K}_R \mathbf{H}_D \boldsymbol{\alpha}^H + \delta_D^2} \\ &s.t. \quad \sum_k |\alpha_k|^2 (|h_{s,k}|^2 P_s + \delta_k^2) \le P_{sum\_\max} \end{aligned} \tag{3-8}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_K]^T \in C^{K\times 1}$ is formed by the coding coefficients, and $\mathbf{H}_D = diag(\mathbf{h}_D) \in C^{K\times K}$. Obviously, it is a non-convex optimization problem. In general, it is very hard to solve it. However, in deep space communication, the number of the relays is always limited. So this optimal problem can be solved directly by Lagrange multiplier, or converted into convex optimization by the method of semi-definite relaxation proposed in [19].

3.2 Performance Analysis

We will give the upper bound of this network model. According to the cut-set bound, we have

$$C_{up} = \max_{\substack{p(x_s,\mathbf{x}):E[|x_s|^2]\le P_s \\ \sum E\left[|x_k|^2\right]\le P_{sum\_\max}}} \min\{I(x_s;\mathbf{y}, y_D \,|\, \mathbf{x}), I(x_s,\mathbf{x}; y_D)\} \tag{3-9}$$

We will calculate the upper bound of the two mutual information seperatively.

$$\begin{aligned} C_{1,up} &= \max_{\substack{p(x_s,\mathbf{x}):E[|x_s|^2]\le P_s \\ \sum E\left[|x_k|^2\right]\le P_{sum\_\max}}} I(x_s;\mathbf{y}, y_D \,|\, \mathbf{x}) \\ &= \max_{p(x_s):E[|x_s|^2]\le P_s} h(\mathbf{y}) - h(\mathbf{z}) \\ &= \frac{1}{2}\log(1 + (\mathbf{h}_s^H \mathbf{K}^{-1} \mathbf{h}_s + \frac{h_{s,D}^2}{\delta_D^2}) P_s) \end{aligned} \tag{3-10}$$

This upper bound can be taken as the capacity of a single input multiple output channel. Hence, we can take the relays as the multiple antenna at the receiver.

$$\begin{aligned} C_{2,up} &= \max_{\substack{p(x_s,\mathbf{x}):E[|x_s|^2]\le P_s \\ \sum E\left[|x_k|^2\right]\le P_{sum\_\max}}} I(x_s,\mathbf{x}; y_D) \\ &= \max_{p(\mathbf{x}):\sum P_{T,k}\le P_{sum\_\max}} h(y_D) - h(z_D) \\ &= \frac{1}{2}\log(1 + \frac{1}{\delta_D{}^2}(h_{s,D}^2 P_s + h_{\max,D}^2 P_{sum\_\max})) \end{aligned} \tag{3-11}$$

where $h_{\max,D} = \max\{h_{1,D}, h_{2,D}, ..., h_{K,D}\}$. That is, the above upper bound is obtained when the relay which has the mimimum path loss is adopted with maximum sum-power. This upper bound can be seen as the capacity of a multiple input single output channel. Then the relays can be taken as the multiple antenna at the source node.

4. Illustrative Example

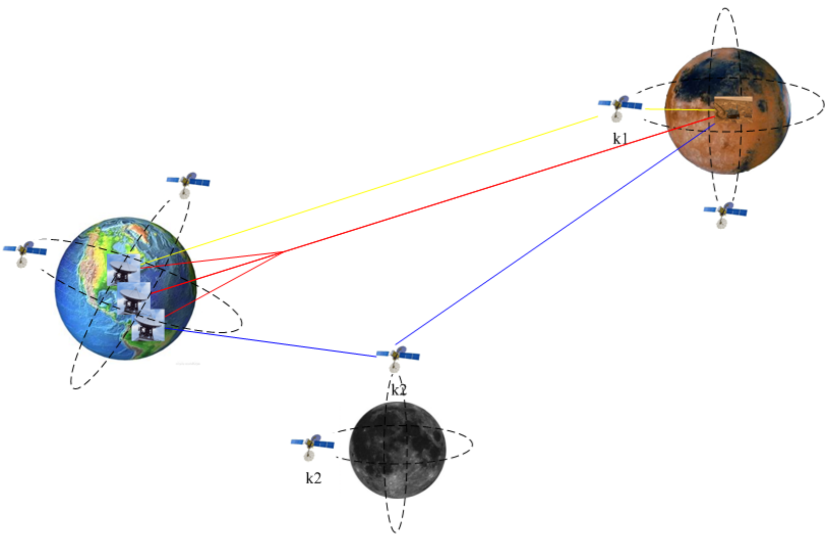

Fig. 3. A dual-hop two-relay deep space communication network from Mars to Earth

To illustrate AFNC scheme, a dual-hop two-relay network is considered in this section for the communication from Mars to Earth, shown in **Fig. 3**, which can be modeled as in **Fig. 4**. S is the source (Mars), D is the receiver (Earth), and $d_l, l = 1,\ldots,5$ is the path length, k_1 and k_2 are relay satellites. There are three paths from S to D. Broadcast is adopted in our approach, that is, the source S broadcast the messages to k_1, k_2 and D, then relays k_1 and k_2 amplify and forward to D. Finally, the messages received at D is the combination of S, k_1 and k_2.

According to (3-1), the channel outputs of the relays are as follows:

$$y_{k_1} = \frac{K_{sh}}{d_1} \cdot \alpha_{k_1} \cdot x + z_{S,k_1} \tag{4-1}$$

$$y_{k_2} = \frac{K_{sh}}{d_2} \cdot \alpha_{k_2} \cdot x + z_{S,k_2} \tag{4-2}$$

where z_{S,k_1}, z_{S,k_2} are the equivalent noise with gaussian distribution after amplifying.

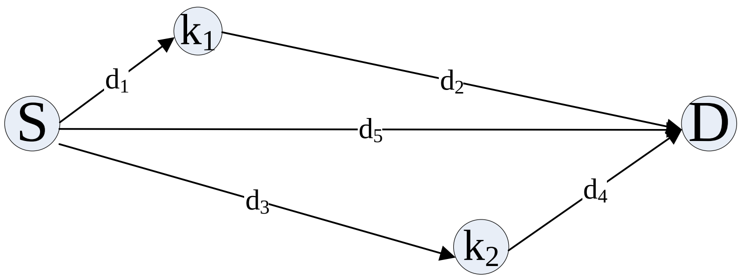

Fig. 4. A dual-hop two-relay network model

Hence, the signal at the receiver is

$$y_D = \left(\frac{K_{sh}}{d_3} \cdot y_{k_1} + z_{k_1,D}\right) + \left(\frac{K_{sh}}{d_5} \cdot y_{k_2} + z_{k_2,D}\right) + \left(\frac{K_{sh}}{d_4} \cdot x + z_{s,D}\right)$$

$$
\begin{aligned}
&= \frac{K_{sh}}{d_3}\cdot(\frac{K_{sh}}{d_1}\cdot\alpha_{k_1}\cdot x + z_{S,k_1}) + z_{k_1,D} + \frac{K_{sh}}{d_5}\cdot(\frac{K_{sh}}{d_2}\cdot\alpha_{k_2}\cdot x + z_{S,k_2}) + z_{k_2,D} + \frac{K_{sh}}{d_4}\cdot x + z_{s,D} \\
&= \frac{K_{sh}{}^2}{d_1 d_3}\cdot\alpha_{k_1}\cdot x + \frac{K_{sh}{}^2}{d_2 d_5}\cdot\alpha_{k_2}\cdot x + \frac{K_{sh}}{d_4}\cdot x + \frac{K_{sh}}{d_3}\cdot z_{S,k_1} + \frac{K_{sh}}{d_5}\cdot z_{S,k_2} + z_{k_1,D} + z_{k_2,D} + z_{S,D} \\
&= (\frac{K_{sh}{}^2}{d_1 d_3}\cdot\alpha_{k_1} + \frac{K_{sh}{}^2}{d_2 d_5}\cdot\alpha_{k_2} + \frac{K_{sh}}{d_4})x + \frac{K_{sh}}{d_3}\cdot z_{S,k_1} + \frac{K_{sh}}{d_5}\cdot z_{S,k_2} + z_{k_1,D} + z_{k_2,D} + z_{S,D} \\
&= (\frac{K_{sh}{}^2}{d_1 d_3}\cdot\alpha_{k_1} + \frac{K_{sh}{}^2}{d_2 d_5}\cdot\alpha_{k_2} + \frac{K_{sh}}{d_4})x + z
\end{aligned}
\tag{4-3}
$$

where the noise is denoted by $z = \frac{K_{sh}}{d_3}\cdot z_{S,k_1} + \frac{K_{sh}}{d_5}\cdot z_{S,k_2} + z_{k_1,D} + z_{k_2,D} + z_{S,D}$, and the terms including x represents the effective signal received at sink node. We assume the noise caused at intermedia nodes are non-coherent, so the total equivalent noise obey Gaussian distribution.

Next, the power received at sink node is

$$
\begin{aligned}
E\left[|y_D|^2\right] &= E[|(\frac{K_{sh}{}^2}{d_1 d_3}\cdot\alpha_{k_1} + \frac{K_{sh}{}^2}{d_2 d_5}\cdot\alpha_{k_2} + \frac{K_{sh}}{d_4})x + z|^2] \\
&= (\frac{K_{sh}{}^2}{d_1 d_3}\cdot h_{S,k_1} + \frac{K_{sh}{}^2}{d_2 d_5}\cdot h_{S,k_2} + \frac{K_{sh}}{d_4})^2 E[|x|^2] + E\left[|z|^2\right] \\
&= (\frac{K_{sh}{}^2}{d_1 d_3}\cdot h_{S,k_1} + \frac{K_{sh}{}^2}{d_2 d_5}\cdot h_{S,k_2} + \frac{K_{sh}}{d_4})^2 P_s + \delta^2
\end{aligned}
\tag{4-4}
$$

where $E[|x|^2] = P_s, E[|z|^2] = \delta^2$.

The equivalent signal is related to the amplification coefficients in our AFNC scheme while the total equivalent noise is only related to network parameters and some constant from expressions (4-3) and (4-4). Since the capacity of the equivalent channel is

$$C = W\log(1+\frac{P}{N_0 W}) \tag{4-5}$$

where W is the bandwidth, N_0 is spectral noise power density, and P is the power of effective signal. For the equivalent Gaussian channel, we concern how to choose the amplification coefficients to maximize the signal-to-noise ratio to achieve the maximum transmission rate, which is equivalent to maximize the power of signal received at sink node here. So the problem is formulated as maximizing the power of signal received at sink node with the sum power over relays limit. That is,

$$
\begin{aligned}
\max \quad & E\left[|y_D|^2\right] \\
s.t. \quad & P_{k_1} + P_{k_2} \le P_{sum\_\max}
\end{aligned}
\tag{4-6}
$$

where $P_{sum\_\max}$ is the maximum sum-power, and

$$P_{k_1}=E\left[|y_{k_1}|^2\right]=(\frac{K_{sh}}{d_1}\cdot\alpha_{k_1})^2P_s+\delta_1^{\ 2},$$
$$P_{k_2}=E\left[|y_{k_2}|^2\right]=(\frac{K_{sh}}{d_2}\cdot h_{S,k_2})^2P_s+\delta_2^{\ 2} \tag{4-7}$$

where $\delta_1^{\ 2}=E[|z_{S,k_1}|^2]$, $\delta_2^{\ 2}=E[|z_{S,k_2}|^2]$.

Lagrange multipliers is need to find the extreme value under the constraint. Let $g(\alpha)=P_{sum\_\max}-\left[(\frac{K_{sh}}{d_1}\cdot\alpha_{k_1})^2+(\frac{K_{sh}}{d_2}\cdot\alpha_{k_2})^2\right]P_s-\delta_1^{\ 2}-\delta_2^{\ 2}$. We define a function $F(\alpha)=E\left[|y_D|^2\right]+\lambda g(\alpha)$ as

$$F(\alpha)=(\frac{K_{sh}^{\ \ 2}}{d_1d_3}\cdot\alpha_{k_1}+\frac{K_{sh}^{\ \ 2}}{d_2d_5}\cdot\alpha_{k_2}+\frac{K_{sh}}{d_4})^2P_s+\delta^2$$
$$+\lambda P_{sum\_\max}-\lambda\left[(\frac{K_{sh}}{d_1}\cdot\alpha_{k_1})^2+(\frac{K_{sh}}{d_2}\cdot\alpha_{k_2})^2\right]P_s-\lambda\delta_1^{\ 2}-\lambda\delta_2^{\ 2} \tag{4-8}$$

Now we get the first-order partial derivatives to the variables α_{k_1} and α_{k_2}, respectively,

$$\frac{\partial F(\alpha)}{\partial\alpha_{k_1}}=[2(\frac{K_{sh}^{\ \ 2}}{d_1d_3}\cdot\alpha_{k_1}+\frac{K_{sh}^{\ \ 2}}{d_2d_5}\cdot\alpha_{k_2}+\frac{K_{sh}}{d_4})\cdot\frac{K_{sh}^{\ \ 2}}{d_1d_3}-2\lambda\frac{K_{sh}^{\ \ 2}}{d_1^{\ 2}}\alpha_{k_1}]P_s$$
$$\frac{\partial F(\alpha)}{\partial\alpha_{k_2}}=[2(\frac{K_{sh}^{\ \ 2}}{d_1d_3}\cdot\alpha_{k_1}+\frac{K_{sh}^{\ \ 2}}{d_2d_5}\cdot\alpha_{k_2}+\frac{K_{sh}}{d_4})\cdot\frac{K_{sh}^{\ \ 2}}{d_2d_5}-2\lambda\frac{K_{sh}^{\ \ 2}}{d_2^{\ 2}}\alpha_{k_2}]P_s \tag{4-9}$$

Then let them equal to 0, i.e.,

$$(\frac{K_{sh}^{\ \ 4}}{d_1^{\ 2}d_3^{\ 2}}-\lambda\frac{K_{sh}^{\ \ 2}}{d_1^{\ 2}})\cdot\alpha_{k_1}+\frac{K_{sh}^{\ \ 4}}{d_1d_2d_3d_5}\cdot\alpha_{k_2}+\frac{K_{sh}^{\ \ 3}}{d_1d_3d_4}=0 \tag{4-10}$$

$$(\frac{K_{sh}^{\ \ 4}}{d_2^{\ 2}d_5^{\ 2}}-\lambda\frac{K_{sh}^{\ \ 2}}{d_2^{\ 2}})\cdot\alpha_{k_2}+\frac{K_{sh}^{\ \ 4}}{d_1d_2d_3d_5}\cdot\alpha_{k_1}+\frac{K_{sh}^{\ \ 3}}{d_2d_4d_5}=0 \tag{4-11}$$

So we have

$$\alpha_{k_1}=\frac{K_{sh}d_1d_3d_5^{\ 2}}{\lambda d_3^{\ 2}d_4d_5^{\ 2}-K_{sh}^{\ \ 2}d_3^{\ 2}d_4-K_{sh}^{\ \ 2}d_4d_5^{\ 2}} \tag{4-12}$$

$$\alpha_{k_2}=\frac{K_{sh}d_2d_3^{\ 2}d_5}{\lambda d_3^{\ 2}d_4d_5^{\ 2}-K_{sh}^{\ \ 2}d_3^{\ 2}d_4-K_{sh}^{\ \ 2}d_4d_5^{\ 2}} \tag{4-13}$$

Substitute (4-12) and (4-13) into the sum-power constraint (4-6), we have

$$\frac{K_{sh}^{\ \ 4}d_3^{\ 4}d_5^{\ 2}+K_{sh}^{\ \ 4}d_3^{\ 2}d_5^{\ 4}}{(\lambda d_3^{\ 2}d_4d_5^{\ 2}-K_{sh}^{\ \ 2}d_3^{\ 2}d_4-K_{sh}^{\ \ 2}d_4d_5^{\ 2})^2}P_s+\delta^2=P_{sum\_\max} \tag{4-14}$$

Then λ is derived from (4-14),

$$\lambda = \frac{\sqrt{\frac{\left(K_{sh}^4 d_3^4 d_5^2 + K_{sh}^4 d_3^2 d_5^4\right) P_s}{P_{sum\_\max} - \delta^2} + K_{sh}^2 d_3^2 d_4 + K_{sh}^2 d_4 d_5^2}}{d_3^2 d_4 d_5^2} \tag{4-15}$$

The two coefficients are obtained as follows by substituting λ into (4-12) and (4-13),

$$\begin{aligned} \alpha_{k_1} &= \frac{K_{sh} d_1 d_3 d_5^2}{\frac{\sqrt{\frac{\left(K_{sh}^4 d_3^4 d_5^2 + K_{sh}^4 d_3^2 d_5^4\right) P_s}{P_{sum\_\max} - \delta^2} + K_{sh}^2 d_3^2 d_4 + K_{sh}^2 d_4 d_5^2}}{d_3^2 d_4 d_5^2} d_3^2 d_4 d_5^2 - K_{sh}^2 d_3^2 d_4 - K_{sh}^2 d_4 d_5^2} \\ &= \frac{K_{sh} d_1 d_3 d_5^2}{\sqrt{\frac{\left(K_{sh}^4 d_3^4 d_5^2 + K_{sh}^4 d_3^2 d_5^4\right) P_s}{P_{sum\_\max} - \delta^2}}} \end{aligned} \tag{4-16}$$

$$\begin{aligned} \alpha_{k_2} &= \frac{K_{sh} d_2 d_3^2 d_5}{\frac{\sqrt{\frac{\left(K_{sh}^4 d_3^4 d_5^2 + K_{sh}^4 d_3^2 d_5^4\right) P_s}{P_{sum\_\max} - \delta^2} + K_{sh}^2 d_3^2 d_4 + K_{sh}^2 d_4 d_5^2}}{d_3^2 d_4 d_5^2} d_3^2 d_4 d_5^2 - K_{sh}^2 d_3^2 d_4 - K_{sh}^2 d_4 d_5^2} \\ &= \frac{K_{sh} d_2 d_3^2 d_5}{\sqrt{\frac{\left(K_{sh}^4 d_3^4 d_5^2 + K_{sh}^4 d_3^2 d_5^4\right) P_s}{P_{sum\_\max} - \delta^2}}} \end{aligned} \tag{4-17}$$

Finally, substitute (4-16) and (4-17) into (4-5), we have

$$E[|\, y_D\,|^2] = \left(\frac{K_{sh}^3 d_5^2 + K_{sh}^3 d_3^2}{\sqrt{\frac{\left(K_{sh}^4 d_3^4 d_5^2 + K_{sh}^4 d_3^2 d_5^4\right) P_s}{P_{sum\_\max} - \delta^2}}} + \frac{K_{sh}}{d_4}\right)^2 P_s + \delta^2 \tag{4-18}$$

So the transmission rate of our AFNC scheme in multiple relay network is

$$\mathrm{C} = W \log\left(1 + \frac{\left(\frac{K_{sh}^3 d_5^2 + K_{sh}^3 d_3^2}{\sqrt{\frac{\left(K_{sh}^4 d_3^4 d_5^2 + K_{sh}^4 d_3^2 d_5^4\right) P_s}{P_{sum\_\max} - N_0 W}}} + \frac{K_{sh}}{d_4}\right)^2 P_s}{N_0 W}\right) \tag{4-19}$$

where $\delta^2 = N_0 W$. And this capacity is achieved when the coefficients are chosen according to (4-12) and (4-13).

5. Simulations

For intuition, we plot the transmission rate of our AFNC scheme in multiple relay network, and compare with that of the direct single link for deep space communication. We made the simulation based on **Fig. 4**. By data query, let $d_1=d_4=3.844\times10^5$km, $d_2=d_3=5.3\times10^7$km,

d_5=5.5×10^7km. The antenna gain is set to be 10, $P_{sum\_max} = 100W$, and spectral noise power density $N_0 = 0.01$. The Ka frequency band is utilized, and let λ=8mm [20]. The transmission rate of AFNC scheme is simulated as the transmit power Ps or the bandwidth W increase respectively, shown in **Fig. 5** and **Fig. 6**.

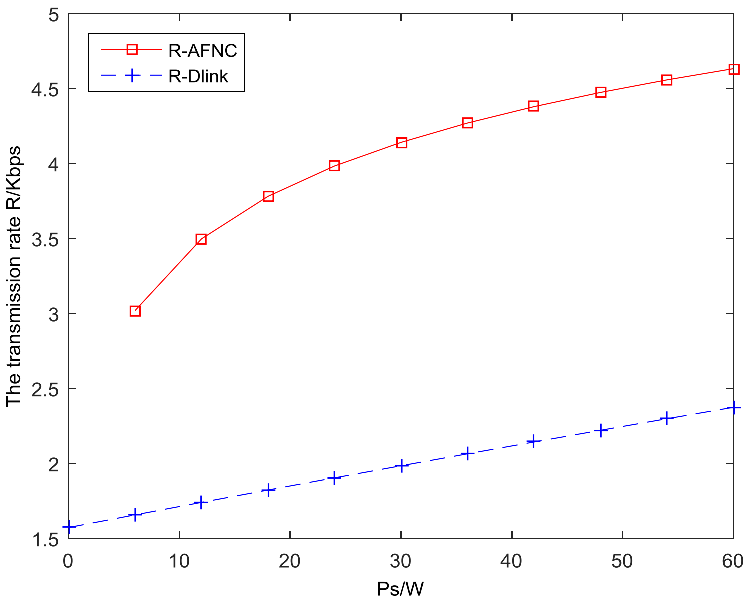

Fig. 5. The transmission rate R increase with the transmit power P_S

In **Fig. 5**, we let the bandwidth W=100MHz, and Ps is in [0,60]w. For fair, we add P_{sum_max} with Ps as the transmit power at the source node for the direct link transmission. That is why the rate is nonzero when Ps is 0. We found that the rate of AFNC scheme has nonzero value after a certain power value because the noise is amplified rather than the signal when the transmit power is very small. In **Fig. 6**, we let Ps =10w, and W is in [50,500]MHz. Through obveration, we found that the result is consistant with theorectic analysis. No matter AFNC scheme or direct link transmission, the transmission rates increase with the bandwidth, and approach to the upper bound of the capacity in terms of SNR. On the other hand, bandwidth is rich for deep space communication. Hence, it is more important for the effect of transmit power on the transmission rate than that of the bandwidth. Furthermore, From both figures, with the same transmit power, the transmission rate R of AFNC scheme in relay network is significantly greater than that of the direct single link channel.

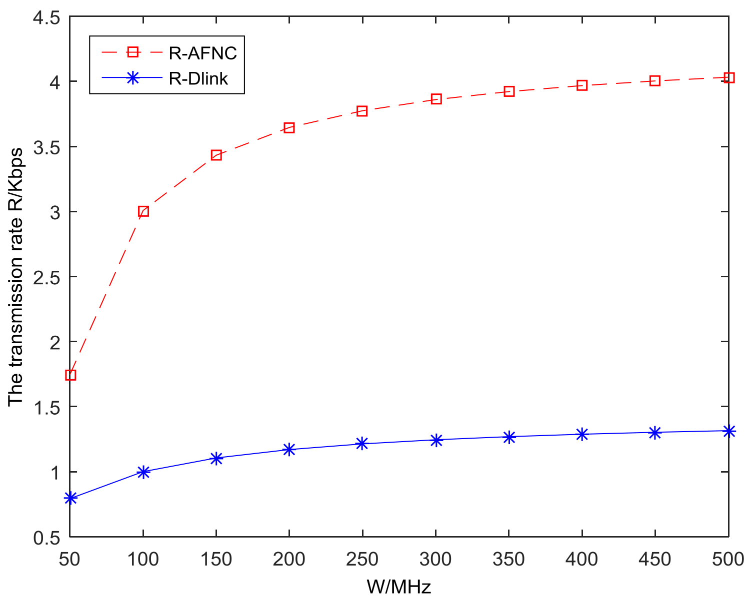

Fig. 6. The transmission rate R increase with the Bandwidth W

6. Conclusion

An AFNC scheme in multi-relay network is proposed to address the bottleneck of deep space communication. In our method, the non-source node could make a combination of its received messages with the coefficients satisfying the sum power constraint over relay space crafts. The optimal amplify coefficients are deduced for a dual-hop two-relay network, and the transmission rate is also given. It is shown that our AFNC scheme is advanced than the single direct link communication through theoretic analysis and simulation. The case of multi-source or multi-hop relay network will be left for feature work, together with the relation between the number of relays and the benefit-cost ratio.

References

[1] Weiren Wu, Guangliang Dong and Haitao Li, "Engineering and Technology of Deep Space TT&C System," *Science Press*, 1st Edition, Jan. 2013. Article (CrossRef Link).

[2] Xianwei Zhou, "Deep Space Communication," *National Defense Industry Press*, Beijing, 5. 2009. Article (CrossRef Link).

[3] Bhasin Kul, De Paula Ramon P., Edwards Charles, "Internet technologies for space-based communications: State of the art and challenges," in *Proc. of 18th International Communications Satellite Systems Conference and Exhibit, American Institute of Aeronautics and Astronautics Inc*, pp. 1-11, 2000. Article (CrossRef Link).

[4] Bhasin K, Hayden J, Agre J R, "Advanced Communication and networking Technologies for Mars exploration," in *Proc. of 19th Annual AIAA International Communications Satellite Systems Conference*, Touloude France, Apr. 2001. Article (CrossRef Link).

[5] Marc Sanchez Net, Inigo del Portillo, Bruce Cameron and Edward Crawley, "Architecting Space Communication Networks under Mission Demand Uncertainty," in *Proc. of 2015 IEEE Aerospace Conference*, Big Sky, America, 2015. Article (CrossRef Link).

[6] Akyildiz I F, Akan O B, Chen C, et.al, "State-of-the-art and research challenges," *IEEE Communication Magazine*, vol. 42, pp.108-118, 2004. Article (CrossRef Link).
[7] Xiao S, Li Y S, Bai B M, "The Key Technologies of Deep Space Communication," *China Communica*, vol. 9, no. 1, pp. 82-94, Dec. 2006. Article (CrossRef Link).
[8] Bisio,I., Lavagetto, F., and Marchese, M., "Introduction to Multi Attribute Decision Making-based Application Layer Joint Coding for Image Transmission over Deep Space Channels," in *Proc.of the Second International Conference on Advances in Satellite and Space Communications (SPACOMM 2010)*, pp. 134-139, Athens, Greece, Jun. 13-19, 2010. Article (CrossRef Link).
[9] Ahswede.R, Cai N, S. Y. R. Li and Yeung R.W., "Network Information Flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204-1216, 2000. Article (CrossRef Link).
[10] Li Shuo-Yen Robert, Sun Qifu Tyler and Shao Ziyu, "Linear Network Coding: Theory and Algorithms," in *Proc. of the IEEE*, vol. 99, no.3, pp.372-387, Mar. 2011. Article (CrossRef Link).
[11] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," *Computer Communication Review*, vol. 37, no. 4, pp. 397-408, Oct. 2007. Article (CrossRef Link).
[12] Seong Hwan Kim, Bang Chul Jung and Dan Keun Sung, "Transmit Power Optimization for Two-Way Relay Channels with Physical-Layer Network Coding," *IEEE Communications Letters*, vol.19, no. 2, pp. 151-154, Feb. 2015. Article (CrossRef Link).
[13] Gu Shushi, Zhang Qinyu, Zhou Jie, "The joint optimaization of digital fountain coding in deep space communication," in *Proc. of the 2011 6th International ICST Conference on Communications and Networking in China (CHINACOM 2011)*, p 71-75, Harbin, China, Aug. 17-19, 2011. Article (CrossRef Link).
[14] J. Li, K. Chen, L. G. Xie, "Self-Cooperative Network Coding in Deep Space Communications," *Applied Mechanics and Materials*, Vol. 662, pp. 267-270, Oct. 2014. Article (CrossRef Link).
[15] Samar Agnihotri, Sidharth Jaggi and Minghua Chen, "Analog network coding in general SNR regime," *IEEE International Symposium on Information Theory*, pp. 2052-2056, 2012. Article (CrossRef Link).
[16] I. Maric and R.D. Yates, "Bandwidth and power allocation for cooperative strategies in Gaussian relay networks," *IEEE Transactions on Information Theory*, vol. 56, no. 4, pp. 1880–1889, 2010. Article (CrossRef Link).
[17] Sburlana S.E., Birnbauma K.M. and Farra W. H., "Deep Space uplink receiver prototype for optical communications," in *Proc. of SPIE – The International Society for Optical Engineering*, vol. 7923, 2011. Article (CrossRef Link).
[18] Zhang Yongfu, Du Jianchao and Xiao Song, "Analysis and Simmulations on Characteristics of Deep Space Channel Model VPAWGN," *Space Electronic Technology*, vol. 7, no. 1, pp.135-142, 2010. Article (CrossRef Link).
[19] Binyue Liu and Ning Cai, "Design of amplify-and-forward relaying schemes for layered relay networks," *IET Commun.*, vol. 8, no. 11, pp. 1910–1918, July 2014. Article (CrossRef Link).
[20] Liang Xianfeng, Zhang Jinzhou, Xie Min, Xie Chunjian, Xiong Weiming, An Junshe, Chen Xiaomin, Sun Huixian, "A Ka band transmitter for deep space communication systems," in *Proc. of 2012 International Conference on microwave and Millimeter Wave Technology, ICMMT 2012 - Proceedings*, vol. 4, pp. 1240-1242. 2012. Article (CrossRef Link).

Wangmei GUO was born in 1984. She received the B.S. degree in information security in Jun. 2006 and Ph.D degree in information and communication engineering in Dec. 2012 both from Xidian University. She is currently a Lecture at Xidian University since Jan. 2013, and his main research interest are network coding and information theory, as well as their applications.

Junhua ZHANG was born in 1982. He is now a engineer in Xi'an Satellite Control Center, and he work in space communication and the applications.

Guiguo Feng was born in 1988. He now is a Ph.D student in Xidian University since Sep.2011, and his interest are information and analog network coding.

Kaijian ZHU was born in 1976. He received the Ph.D degree in 2009 from TsingHua University. He was a visting scholar in school of Civil and Earth Science at Newcastle University in the United Kingdom from Dec. 2013 to Dec. 2014. He is now a senior engineer in State key laboratory of Astronautic Dynamics, Xi'an Satellite Control Center. His current research interst includes space communication and interplanetary orbit design.

Jixiang Zhang was born in 1992. He received the B.S. degree in communication engineering from Northwest University in 2013. He now is a Master student in Xidian University since Sep. 2013, and his research interst are information and optimalization.