An Induced Hesitant Linguistic Aggregation Operator and Its Application for Creating Fuzzy Ontology

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Received March 5, 2018; revised April 17, 2018; accepted May 18, 2018; published October 31, 2018

Abstract

An induced hesitant linguistic aggregation operator is investigated in the paper, in which, hesitant fuzzy linguistic evaluation values are associated with probabilistic information. To deal with these hesitant fuzzy linguistic information, an induced hesitant fuzzy linguistic probabilistic ordered weighted averaging (IHFLPOWA) operator is proposed, monotonicity, boundary and idempotency of IHFLPOWA are proved. Then andness, orness and the entropy of dispersion of IHFLPOWA are analyzed, which are used to characterize the weighting vector of the operator, these properties show that IHFLPOWA is extensions of the induced linguistic ordered weighted averaging operator and linguistic probabilistic aggregation operator. In this paper, IHFLPOWA is utilized to gather linguistic information and create fuzzy ontologies, and a movie fuzzy ontology as an illustrative case study is used to show the elaboration of the proposed method and comparison with the existing linguistic aggregation operators, it seems that the IHFLPOWA operator is an useful and alternative operator for dealing with hesitant fuzzy linguistic information with probabilistic information.

Keywords: Hesitant fuzzy linguistic terms set, 2-tuple linguistic model, linguistic multi-criteria group decision making, fuzzy ontology, linguistic probabilistic aggregation operator.

http://doi.org/10.3837/tiis.2018.10.018 **ISSN : 1976-7277**

1. Introduction

Decision making is a cognitive process based on different mental and reasoning processes that lead to the choice of a suitable alternative from a set of possible alternatives in a decision situation [1–3]. Due to the inherent complexity and uncertainty of the decision situation or the existence of multiple and conflicting objectives, human beings often use fuzzy linguistic terms to express complex or uncertain information in decision making process, *i.e.*, assessments of possible alternatives in a decision situation are expresses by fuzzy evaluation linguistic terms, such as comfortable degree of vehicles are often expressed by consumers using linguistic terms "*poor, fair, good, perfect*". Up to now, linguistic decision making has been widely used in many applications, such as in [4], linguistic decision making has been utilized to extract users' information and create fuzzy ontologies for storing and sharing people knowledge automatically.

Theoretically, the core of linguistic decision making is fuzzy linguistic information processing method, the remarkable works of fuzzy linguistic information processing was initially proposed by Zadeh in [5], which are rooted on linguistic variable and fuzzy set [6], a linguistic variable is a variable whose values are not numbers, but words or sentences in a natural or artificial language. Linguistic variables provide us a flexible and reliable form to represent qualitative information in decision makings, however, the approach has some limitations due to using membership functions, such as a priori linguistic terms set, limitation of the number of linguistic terms, computational complexity, a lack of accuracy, and loss information in the approximation processes [7]. To overcome these weaknesses, the 2-tuple linguistic model has been proposed in [7], which provides a continuous fuzzy representation for linguistic terms by the translation of the linguistic term obtained from the symbolic computation to the closest linguistic term in the initial linguistic terms set. Up to now, the 2-tuple linguistic model has become a useful and fundamental tool for expressing and handling fuzzy linguistic information, and many new 2-tuple linguistic-inspired models have been developed and used to handle many different real-world decision making [1], such as, Xu [2] introduced the extended linguistic variable based on the concept of virtual linguistic values to improve the operational laws of symbolic operations. Wang and Hao [8] proposed the linguistic proportional 2-tuple model to represent linguistic information that is a generalization and extension of the 2-tuple linguistic model, Guo et al.[9] extended the linguistic proportional 2-tuple model by using a third parameter to deal with incomplete linguistic preferences. Dong et al. [10] presented the concept of numerical scale with the aim of completing the 2-tuple linguistic model and proportional 2-tuple models and making the elicitation of information more consistent in different decision situations. Li [11] proposed an extended 2-tuple linguistic model that fuses the use of virtual linguistic values and 2-tuple linguistic values. Wei [12] investigated the 2-tuple linguistic multiple attribute group decision making problems in which the information about attribute weights is partially known. Yang [13] developed the counted linguistic variable for representing and aggregating linguistic information with the aim of providing better results and being easier to understand. Cables et al. [14] proposed a decision making method in which the decision makers provided their assessment information to represent their qualitative preferences under 2-tuple linguistic environment. Moreover, many 2-tuple linguistic aggregation operators have been proposed in literatures [15–25] to make the aggregation of linguistic information much more flexible, we refer [1, 7] for more details about 2-tuple linguistic model and decision makings based on 2-tuple linguistic model. Motivated by hesitant fuzzy sets [26] and 2-tuple linguistic model, the concept of hesitant fuzzy linguistic term sets (HFLTSs) have been proposed in [27] to deal with hesitant fuzzy linguistic information in linguistic decision makings. HFLTSs have attracted many scholars' attention since its appearance and some research results have been presented [28–30], for example, Wei, et al. [31] defined two aggregation operators for HFLTSs: a hesitant fuzzy LWA operator and a hesitant fuzzy LOWA operator after defining operations on HFLTSs and possibility degree formulas for comparing HFLTSs. Wang, et al. [32] proposed a new approach to solve multi-criteria group decision-making problems in which the criteria are in different priority levels and the criteria values take the form of interval-valued hesitant fuzzy linguistic numbers. Chen, et al. [33] presented a new method for multi-criteria group decision making based on hesitant fuzzy linguistic term sets using the pessimistic attitude and the optimistic attitude of the decision maker. Wang, et al. [34] proposed an outranking approach for multi-criteria group decision-making with hesitant fuzzy linguistic term sets. Liao, et al. [35] developed different types of distance and similarity measures for HFLTS, based on which an approach for multi-criteria decision making problem is given. Liu, et al. [36] presented a new representation of HFLTS by means of a fuzzy envelope and applied it to multi-criteria group decision making. Zhang [37] presented hesitant fuzzy power aggregation operators for multi-criteria group decision making. Zhang, et al. [38] proposed an approach to solve the problem of ranking alternatives expressed with HFLTSs, four uncertain hesitant fuzzy linguistic aggregation operators and an approach to hesitant fuzzy linguistic multiple attribute decision making. Lee, et al. [39] devised a new fuzzy decision making method and a new fuzzy group decision making method based on the proposed likelihood-based comparison relations of HFLTSs and the four proposed aggregation operators of HFLTSs.

Probabilistic information have been also widely considered in decision making, especially, aggregation operators with probabilistic information have been widely studied, such as Wu [40] provided a possibility distribution based approach to carry out the aggregation process of HFLTSs. Zeng, et al. [41] used distance measures in a unified framework between the probability and the ordered weighted averaging (OWA) operator to present the uncertain probabilistic OWA distance operator and applied it to a group decision making problem. Merig´o, et al. [42] presented the probabilistic ordered weighted averaging distance (POWAD) operator that uses a unified model between probabilities and the OWA operator considering the degree of importance that each concept has. Merig´o, et al. [43] presented the induced linguistic probabilistic ordered weighted average (ILPOWA) operator using probabilities and the OWA operator in the same formulation and developed a new approach for linguistic group decision making based on the ILPOWA operator. Merig´o, et al. [44] introduced the uncertain generalized probabilistic weighted averaging (UGPWA) operator unifying the probability and the weighted average in the same formulation and analyzed its applicability in a group decision making problem. Merig´o [45] introduced the probabilistic OWA (POWA) operator considering the degree of importance that the probability and the OWA operator have in the aggregation and analyzed its applicability in group decision making.

In many real world practices, when a decision maker is asked to provide his/ her preference on an alternative with respect to a criterion, there exists the following case: on the one hand, the decision maker may be hesitant between several linguistic terms; on the other hand, based on his/her knowledge, experience, cultural foundation, and educational background, the decision maker may prefer to use ones of the linguistic terms than others to evaluate the alternative, for example, suppose a set of nine linguistic terms be

$$S=\left\{\begin{array}{l} s_0 = \textit{extremely low } (EL),\ s_1 = \textit{very low}(VL),\ s_2 = \textit{low } (L), \\ s_3 = \textit{slightly low } (SL),\ s_4 = \textit{medium } (M),\ s_5 = \textit{slightlyhigh } (SH), \\ s_6 = \textit{high } (H),\ s_7 = \textit{very high } (VH),\ s_8 = \textit{extremely high}(EH) \end{array}\right\}.$$

On the one hand, the decision maker might be hesitant between the linguistic terms "*H*" and "*VH*" when he/ she evaluates an alternative with respect to a criterion; On the other hand, due to his/ her knowledge, experience, cultural foundation, and educational background, the decision maker may prefer to use the linguistic term "*VH*" than "*H*" to evaluate the alternative. In such case, only the HFLTS $\{s_6, s_7\}$ cannot sufficiently reflect the decision maker's opinion, combing the HFLTS and probabilistic information maybe a good idea, *i.e.*, $\{(s_6, 0.3), (s_7, 0.7)\}$, where 0.7 and 0.3 indicate the possibility of "*V H*" and "*H*" used by the decision maker, respectively. In real world practices, $\{(s_6, 0.3), (s_7, 0.7)\}$ means that the decision maker is hesitant between s_6 and s_7 as well as the decision maker prefers to use s_7 (with possibility 0.7) than s_6 (with possibility 0.3). Accordingly, the major contributions of the paper are summarized as follows:

(1) Hesitant fuzzy linguistic term sets with probabilistic information is formalized in this paper, which can be used to express evaluation linguistic information with hesitancy and preference;

(2) An induced hesitant fuzzy linguistic probabilistic ordered weighted averaging (IHFLPOWA) operator is proposed to aggregate hesitant fuzzy linguistic term sets with probabilistic information and properties of the IHFLPOWA operator are analyzed, which show that the IHFLPOWA operator is an extension of several existed linguistic aggregation operators;

(3) The IHFLPOWA operator is utilized to create a movie fuzzy ontology, in which evaluations of alternatives (movies) are hesitant fuzzy linguistic term sets with probabilistic information provided by Internet users.

The rest of this paper is organized as follows: In Section 2, we briefly review 2-tuple linguistic model, hesitant fuzzy linguistic term sets, several basic linguistic aggregation operators and fuzzy ontology. In Section 3, we propose the IHFLPOWA operator and analyze some different types of the IHFLPOWA operator by considering particular cases of the weighting vector. In Section 4, inspired by interesting works on fuzzy ontology [4], a movie's fuzzy ontology is provided to show feasibility and efficiency of the IHFLPOWA operator in extract users' hesitant fuzzy linguistic and probabilistic information and create knowledge databases for storing and sharing people knowledge automatically. Section 5 concludes this paper.

2. Preliminaries

Formally, 2-tuple linguistic model [5] is described as follows: Let $S=\{s_0, s_1, \cdots, s_g\}$ be a set of linguistic terms and $\beta \in [0, g]$ be a value supporting the result of a symbolic aggregation operation. Then a 2-tuple linguistic value that expresses the equivalent information to β is obtained with the function $\Delta : [0, g] \rightarrow \overline{S}$ such that $\Delta(\beta) = (s_i, \alpha)$ with $i = round(\beta)$ and $\alpha = \beta - i \in [-0.5, 0.5)$, where s_i has the closest index label to β and α is the value of the symbolic translation, $round(\cdot)$ is the usual round operation. The ordering of linguistic information is processed by the linear ordered structure of linguistic values, and their natural number indexes are used to explain the ordering, *i.e.*, for any 2-tuple linguistic values (s_i,α_i) and (s_j,α_j), $(s_i,\alpha_i)\le(s_j,\alpha_j)$ if and only if $\Delta^{-1}(s_i,\alpha_i)=i+\alpha_i\le\Delta^{-1}(s_j,\alpha_j)=j+\alpha_j$ all linguistic 2-tuples are denoted by $\overline{S}=\{(s_i,\alpha_i)|s_i\in S,\alpha\in[-0.5,0.5)\}$.

Hesitant fuzzy set on X is a function h that returns a subset of values in [0,1], where X is a reference set [26]. A hesitant fuzzy set h_M associated to M is defined as $h_M(x)=\bigcup_{\mu_i\in M}\{\mu_i(x)\}$, where $M=\{\mu_1,\ \mu_2,\cdots\mu_n\}$ be a set of membership functions. Given a hesitant fuzzy set h, its lower bound $h^-(x)$,upper bound $h^+(x)$ and complement h^c are defined as $h^-(x)=\min h(x)$, $h^+(x)=\max h(x)$ and $h^c=\bigcup_{\gamma\in h}\{1-\gamma\}$, respectively.

2.1 Hesitant Fuzzy Linguistic Term Sets

HFLTSs have been proposed to deal with the situations where decision makers have hesitancy in providing their linguistic preferences over objects [27]. Formally, an HFLTS, H_S, is an ordered finite subset of the consecutive linguistic terms of $S=\{s_0,s_1,\cdots,s_g\}$, such as $H_S=\{s_3,s_4,s_5\}$. Some of its basic operations of hesitant fuzzy linguistic information are briefly viewed as follows.

— Lower bound: $H_{S^-}=\min(s_i)=s_j, s_i\in H_S$ and $s_i\ge s_j\forall i$;

—Upper bound: $H_{S^+}=\max(s_i)=s_j, s_i\in H_S$ and $s_i\le s_j\forall i$;

—Complement: $H_S^c=S-H_S=\{s_i|s_i\in S$ and $s_i\not\subset H_S\}$;

—Union: $H_S^1\bigcup H_S^2=\{s_i|s_i\in H_S^1$ or $s_i\in H_S^2\}$;

—Intersection: $H_S^1\bigcap H_S^2=\{s_i|s_i\in H_S^1$ and $s_i\in H_S^2\}$;

—Envelope: $env(H_S)=[H_{S^-},H_{S^+}]$.

The envelope of a HFLTS is a linguistic interval whose limits are obtained by means of upper bound (max) and lower bound (min). Based on the envelopes of HFLTSs, the follows can be defined [27]

$H_S^1(\theta)>H_S^2(\theta)$ if and only if $env(H_S^1(\theta))>env(H_S^2(\theta))$,

$H_S^1(\theta)=H_S^2(\theta)$ if and only if $env(H_S^1(\theta))=env(H_S^2(\theta))$.

Accordingly, the comparison among HFLTSs can be converted into the comparison among their envelopes.

2.2 Linguistic or HFLTSs Aggregation Operators

Here, we briefly review several important linguistic or HFLTSs aggregation operators, which are widely used in linguistic decision making problems.

– The LWA operator [46]: A LWA operator of dimension n is a mapping, LWA: $S^n\to\overline{S}$, that has an associated vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\mathrm{LWA}(s_{\alpha_1}, s_{\alpha_2}, \cdots, s_{\alpha_n}) = \Delta\left(\sum_{i=1}^{n} w_i \Delta^{-1}(s_{\alpha_i}, 0)\right), \quad (1)$$

where s_{α_i} $(i = 1, 2, \cdots, n)$ are the aggregated values.

– The LOWA operator [47]: A LOWA operator of dimension n is a mapping LOWA: $S^n \to \bar{S}$, which has an associated weighting vector W such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.Furthermore,

$$\mathrm{LOWA}(s_{\alpha_1}, s_{\alpha_2}, \cdots, s_{\alpha_n}) = \Delta\left(\sum_{j=1}^{n} w_j \Delta^{-1}(s_{\beta_j}, 0)\right), \quad (2)$$

where s_{β_j} is the jth largest element in $\{s_{\alpha_1}, s_{\alpha_2}, \cdots, s_{\alpha_n}\}$.

– The IULOWA operator [48]: An induced uncertain linguistic OWA (IULOWA) operator is a mapping as follows

$$\mathrm{IULOWA}(\langle u_1, \tilde{s}_1\rangle, \cdots, \langle u_n, \tilde{s}_n\rangle) = w_1 \tilde{s}_{\beta_1} \oplus \cdots \oplus w_n \tilde{s}_{\beta_n}, \quad (3)$$

where $w = (w_1, w_2, \cdots, w_n)$ is a weighting vector, such that $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, $\tilde{s}_{\beta_j}$ is the value $\tilde{s}_i$ of the pair $\langle \boldsymbol{u}_i, \tilde{s}_i \rangle$ having the jth largest u_i, and u_i in $\langle \boldsymbol{u}_i, \tilde{s}_i \rangle$ is referred as the order inducing variable and $\tilde{s}_i$ as the uncertain linguistic argument variable, *i.e.*, $\tilde{s}_i = [s_{\alpha_i}, s_{\beta_i}](\alpha_i < \beta_i, s_{\alpha_i}, s_{\beta_i} \in S)$.

– The ILPOWA operator [43]: An ILPOWA operator of dimension n is a mapping ILPOWA: $S^n \times S^n \to S$ with an associated weighting vector W of dimension n and probability P such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, $\sum_{i=1}^{n} p_i = 1$, $p_i \in [0,1]$, respectively, i.e.,

$$\mathrm{ILPOWA}(\langle u_1, s_{\alpha_1}\rangle, \cdots, \langle u_n, s_{\alpha_n}\rangle) = \Delta\left(\sum_{j=1}^{n} \breve{v}_j \Delta^{-1}(s_{\beta_j}, 0)\right), \quad (4)$$

where s_{β_j} is the jth largest linguistic value s_{α_i} induced by u_i, $\breve{v}_j = \beta w_j + (1-\beta) p_j$ with $\beta \in [0,1]$ and p_j is the probability of s_{α_i} induced by u_i. Due to $\breve{v}_j = \beta w_j + (1-\beta) p_j$, the ILPOWA operator can also be rewritten as follows

$$\mathrm{ILPOWA}(\langle u_1, s_{\alpha_1}\rangle, \cdots, \langle u_n, s_{\alpha_n}\rangle) = \Delta\left(\beta \sum_{j=1}^{n} w_j \Delta^{-1}(s_{\beta_j}, 0) + (1-\beta) \sum_{i=1}^{n} p_i \Delta^{-1}(s_{\alpha_i}, 0)\right), \quad (5)$$

Based on the above mentioned linguistic aggregation operators, we notice that the LOWA operator defined by (2) is an extension of the LWA operator defined by (1) if s_{β_j} is actually equal to s_{α_j}, the IULOWA operator defined by (3) is an extension of the LOWA operator defined by (2) if $s_{\alpha_i} = s_{\beta_i}$ for any $\tilde{s}_i = [s_{\alpha_i}, s_{\beta_i}]$ and the order induced by u_i is actually equal to the order induced by $\tilde{s}_i$. Formally, if $\beta = 1$, then the ILPOWA operator defined by (4) or (5) reduces to the induced linguistic OWA operator. If $\beta = 0$, then the ILPOWA operator defined by (4) or (5) reduces to the linguistic probabilistic aggregation operator.

Several aggregation operators of HFLTSs have been proposed as follows:

– The HLWA operator [31]: Let $S = \{s_0, \cdots, s_g\}$ be a linguistic term set, $H_S^1, \cdots, H_S^n$ be n HFLTSs on S. Let $w = (w_1, w_2, \cdots, w_n)^T$ be a weighting vector of $H_S^j (j = 1, \cdots, n)$ with $w_j \geq 0 (j = 1, \cdots, n)$ and $\sum_{j=1}^n w_j = 1$ .Then, the hesitant fuzzy linguistic weighted aggregation (HLWA) operator is defined as

$$\mathrm{HLWA}(H_S^1, \cdots, H_S^n) = C^n \{w_k, H_S^k, k = 1, 2, \cdots, n\}$$
$$= w_1 \odot H_s^1 \oplus (1 - w_1) \odot C^{n-1} \left\{ \frac{w_h}{\sum_{k=2}^n w_k}, H_s^h, h = 2, \cdots, n \right\}. \quad (6)$$

– The HLOWA operator [31]: Let S, $H_S^i (i = 1, \cdots, n)$ be as before. The hesitant fuzzy LOWA (HLOWA) operator is defined as

$$\mathrm{HLOWA}(H_S^1, \cdots, H_S^n) = C^n \{w_k, H_s^{\sigma_k}, k = 1, 2, \cdots, n\}$$
$$= w_1 \odot H_S^{\sigma_k} \oplus (1 - w_1) \odot C^{n-1} \left\{ \frac{w_h}{\sum_{k=2}^n w_k}, H_S^{\sigma_h}, h = 2, \cdots, n \right\}, \quad (7)$$

where $w = (w_1, w_2, \cdots, w_n)^T$ is an associated weighting vector of the operator with $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$; $(H_S^{\sigma_1}, \cdots, H_S^{\sigma_n})$ is a permutation of $H_S^1, \cdots, H_S^n$ such that $H_S^{\sigma_i} > H_S^{\sigma_j}$ or $H_S^{\sigma_i} \triangleright H_S^{\sigma_j}$ for all $i < j$.

– The UHFLOWA operator [38]: An uncertain hesitant fuzzy linguistic ordered weighted averaging operator UHFLOWA: $\tilde{S}^n \to \tilde{S}$ is defined as

$$\mathrm{UHFLOWA}(H_S^1, H_S^2, \cdots, H_S^n) = w_1 \tilde{H}_S^1 \oplus \cdots \oplus w_n \tilde{H}_S^n, \quad (8)$$

where $\tilde{S}$ is the set of all HFLTSs, $\tilde{H}_S^i (i = 1, 2, \cdots, n)$ is the i th largest of $\tilde{H}_S^j (j = 1, 2, \cdots, n)$ and $w = (w_1, w_2, \cdots, w_n)^T$ is weights.

– The UHFLWA operator [38]: An uncertain hesitant fuzzy linguistic weighted averaging operator UHFLWA: $\tilde{S}^n \to S^n$, is defined as

$$\mathrm{UHFLWA}(H_S^1, H_S^2, \cdots, H_S^n) = \omega_1 H_S^1 \oplus \cdots \oplus \omega_n H_S^n, \quad (9)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weighting vector of hesitant fuzzy linguistic variables $\tilde{H}_S^i (i = 1, 2, \cdots, n)$ $\sum_{i=1}^n \omega_i = 1$ and $\omega_i \geq 0$.

– The UHFLHA operator [38]: An uncertain hesitant fuzzy linguistic hybrid aggregation operator UHFLHA: $\tilde{S}^n \to \tilde{S}$ is defined as

$$\text{UHFLH}A_{\omega,\ w} = w_1\overline{H}_S^1 \oplus \cdots \oplus w_n\overline{H}_S^n, \tag{10}$$

where $\overline{H}_S^i$ is the i th largest of $\dot{H}_S^j \left(\dot{H}_S^j = n\omega_j H_S^j, j = 1,\cdots,n\right)$, $w = [w_1, w_2, \cdots, w_n]^T$ and $\omega = [\omega_1, \omega_2, \cdots, \omega_n]^T$ satisfy $0 \le w_i, \omega_i \le 1$ $(i = 1,\cdots,n)$, $\sum_{i=1}^n w_i = 1$ and $\sum_{i=1}^n \omega_i = 1$.

It is obvious that if $w_i = \frac{1}{n}(i = 1, \cdots, n)$, then the UHFLHA operator defined by (10) is reduced to the UHFLWA operator defined by (9). If $\omega_i = \frac{1}{n}(i = 1, \cdots, n)$, then the UHFLOWA operator defined by (8) is reduced to the HLOWA operator defined by (7) .

2.3 Fuzzy ontology

On the one hand, the Internet has become a place where users can connect and share large amounts of information, this means that Internet users have become providing and consuming information entities and information is more accessible and available than ever. On the other hand, in most cases, the Internet information is little used by users because it is badly structured. It seems that ontologies [53] are tools that provide a way of sorting, classifying and describing large amounts of Internet information, and knowledge databases created using ontologies are easy to manage and allow users to search for information and extract conclusions [4]. Theoretically, crisp ontologies allow each element to be described or not by each concept in the ontology, however in the Internet environments, users often provide imprecision conceptual information, this means that each element is described by each concept using a particular degree in the interval $[0, 1]$ or linguistic terms such as Low, Medium or High, fuzzy ontologies as tools are used to solve the such problem. Formally, a fuzzy ontology [54] is a quintuple $O = \{I, C, R, F, A\}$, where I is a set of individuals, C is a set of concepts, R is the set of relations, F is a set of fuzzy relations which allow individuals to be related to concepts or other individuals to a certain degree and A is the set of axioms. In crisp ontologies, each individual is related or not to each concept or individual, *i.e.*, $\{0, 1\}$. In fuzzy ontologies, individuals can establish relations in a fuzzy way, such as using a membership function, the interval $[0, 1]$ or linguistic terms [5,6].

The method to create automatic knowledge ontology using information from Internet users includes the following four steps [4]:

1. Individuals and concepts definition: Each designed fuzzy ontology is related to a certain topic. Therefore, first, it is necessary to identify the individuals and concepts that are related with the topic that is being dealt with and the relations among the different elements that the fuzzy ontology is comprised of. In this paper, it is considered that every individual is related to every concept. Also, it is assumed that individuals are not related.

2. Ranking process: Group decision making processes are used in order for Internet users to be able to define the values of the relations between each individual and concept.

3. Fuzzy ontology creation process: Once the relation values between each individual and concept are defined, the fuzzy ontology can be created by gathering the information. As a result, the knowledge that has been provided by the users is stored in an organized way. Also, other Internet users can access and benefit from it. Intuitively, the fuzzy ontology creation process follows ranking linguistic term set association and fuzzy ontology structure

construction, ranking linguistic term set association means that a $S=\{s_0, s_1, \cdots, s_g\}$ containing the same number of labels as individuals in the fuzzy ontology is defined. The label indicating the highest value is assigned to the first individual in the ranking, the second highest value to the second position in the ranking and so on. After applying the ranking linguistic term set association to all the concepts in the fuzzy ontology, the information is gathered and the fuzzy ontology is constructed.

4. Fuzzy ontology consulting process: The step is followed so that Internet users can retrieve information.

In this paper, we select a movie fuzzy ontology as an illustrative case study (Section 4) to show creating fuzzy ontology based on linguistic aggregation operator, in which, each individual related to each concept is described by hesitant fuzzy linguistic term sets on $S=\{s_0=EL, s_1=VL, s_2=L, s_3=SL, s_4=M, s_5=SH, s_6=H, s_7=VH, s_8=EH\}$ with probabilistic information, and the induced hesitant fuzzy linguistic probabilistic OWA operator is used to gather the hesitant fuzzy linguistic information and create the movie fuzzy ontology.

3. The Induced Hesitant Fuzzy Linguistic Probabilistic OWA Operator

In this section, we firstly consider hesitant fuzzy linguistic term sets with probabilistic information provided by decision makers, then we propose an induced hesitant fuzzy linguistic probabilistic OWA (IHFLPOWA) operator to deal with hesitant fuzzy linguistic term sets with probabilistic information and analyze several properties of the IHFLPOWA operator.

3.1 The IHFLPOWA Operator

Let $S=\{s_0, s_1, \cdots, s_g\}$ be an initial linguistic term set. A hesitant fuzzy linguistic term set with probabilistic information can be formalized as $H^P=\{(s_{\alpha_1}, p_1), \cdots, (s_{\alpha_n}, p_n)\}$, where $\sum_{i=0}^{g} p_i = 1$ and $p_i \in [0,1]$, all H^P on S can be denoted as

$$L_{hp}=\left\{H^P=\{(s_{\alpha_1}, p_1), \cdots, (s_{\alpha_n}, p_n)\} \mid \forall H^P, s_{\alpha_i} \in S, \sum_{i=0}^{n} p_i = 1\ p_i \in [0,1], 1 \le n \le g+1\right\}.$$

In practices, H^P can be explained from the following two aspects: One is that H^P expresses hesitancy of decision makers, the other is that each element s_{α_i} in H^P has a probability p_i according to the habits of language use, perceptions for linguistic values and knowledge background of decision makers, or the decision maker prefer to use s_{α_i} as evaluation linguistic value than $s_{\alpha_{i'}}$ if $p_i \ge p_{i'}$. To deal with H^P in decision making, we propose the following IHFLPOWA operator.

An IHFLPOWA operator of dimension n is a mapping IHFLPOWA: $(L_{hp})^n \times (L_{hp})^n \to \bar{S}$ that has an associated weighting vector W of dimension n such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, *i.e.*,

$$\text{IHFLPOWA}\left(\langle u_1, H_1^P\rangle, \cdots, \langle u_n, H_n^P\rangle\right) = \Delta\left(\sum_{j=1}^{n} w_j \Delta^{-1}\left(H_j^P\right)\right) = \Delta\left(\sum_{j=1}^{n} w_j \left(\sum_{k=1}^{l_j} p_{kj} \Delta^{-1}\left(s_{kj}\right)\right)\right), \tag{11}$$

in which, for any $i \in \{1,\cdots,n\}$, u_i is the order inducing variable of $H_i^P \in L_{hp}$, *i.e.* H_j^P is the hesitant fuzzy linguistic term set H_i^P with probabilistic information in $\{H_i^P \mid H_i^P = \{(s_{1i}, p_{1i}), \cdots, (s_{l_i i}, p_{l_i i}) \in L_{hp}, i = 1,\cdots,n\}$ induced by the jth largest u_i.

Example 1 Let a linguistic term set be $S = \{s_0, \cdots, s_8\}$. Assume the following hesitant fuzzy linguistic term sets with probability information in the aggregation process: $H_1^P = \{(s_2, 0.5), (s_3, 0.3), (s_4, 0.2)\}$, $H_2^P = \{(s_6, 0.6), (s_7, 0.4)\}$, $H_3^P = \{(s_5, 1)\}$, $H_4^P = \{(s_4, 0.5), (s_5, 0.2), (s_6, 0.2), (s_7, 0.1)\}$. Referring to [43], we assume the weighting vector $w = (w_1, w_2, w_3, w_4) = (0.2, 0.2, 0.3, 0.3)$ and the order inducing variables $U = (u_1, u_2, u_3, u_4) = (8, 4, 9, 6)$. According to the probabilistic weighting vector $P_i\ (i = 1,2,3,4)$ of each hesitant fuzzy linguistic term set, we know the decision maker holds a pessimistic attitude towards this aggregation process. Using the IHFLPOWA operator defined by (11) to aggregate the existed information, we have

1. Reorder these H_i^P according to the order inducing variable $u_i\ (i = 1,2,3,4)$, we have $u_3 > u_1 > u_4 > u_2$ and thus the order of H_i^P is listed as $H_3^P > H_1^P > H_4^P > H_2^P$.
2. Aggregate the linguistic values $s_{ki}(k = 1, \cdots, l_i)$ in each hesitant fuzzy linguistic term set $H_i^P\ (i \in 1,\cdots,n)$ with the jth largest u_i and obtain the result $\Delta^{-1}(H_j^P)(j = 1,2,3,4)$ as follows $\Delta^{-1}(H_1^P) = 1 \times \Delta^{-1}(S_5) = 1 \times 5 = 1$,
$\Delta^{-1}(H_2^P) = 0.5 \times \Delta^{-1}(s_2) + 0.3 \times \Delta^{-1}(s_3) + 0.2 \times \Delta^{-1}(s_4) = 2.7$,
$\Delta^{-1}(H_3^P) = 0.5 \times \Delta^{-1}(s_4) + 0.2 \times \Delta^{-1}(s_5) + 0.2 \times \Delta^{-1}(s_6) + 0.1 \times \Delta^{-1}(s_7) = 4.9$
and $\Delta^{-1}(H_4^P) = 0.6 \times \Delta^{-1}(s_6) + 0.4 \times \Delta^{-1}(s_7) = 6.4$.
3. IHFLPOWA
$$= \Delta\left(\sum_{j=1}^{4} w_j \Delta^{-1}(H_j^P)\right) = \Delta(0.2 \times 5 + 0.2 \times 2.7 + 0.3 \times 4.9 + 0.3 \times 6.4) = (s_5, -0.07).$$

Note that in the case of ties in the reordering process of the inducing variables [43], we recommend the methodology provided by Yager [49] that replaces these tied arguments by their average.

Theorem 1 The IHFLPOWA operator defined by (11) is monotonic, bounded and idempotent.

Proof Based on (11), we have

1) Monotonicity: For any i, let $H_i^P = \{(s_{1i}, p_{1i}), \cdots, (s_{l_i i}, p_{l_i i})\}$ and $H_i^{P'} = \{(s'_{1i}, p'_{1i_i}), \cdots, (s'_{li_i}, p'_{li_i})\}$. If $s_{ki} \le s'_{ki}, p_{ki} = p'_{ki}, \forall k, i$, then $\sum_{k=1}^{l_i} p_{ki}\Delta^{-1}(s_{ki}) \le \sum_{k=1}^{l_i} p'_{ki}\Delta^{-1}(s'_{ki})$, *i.e.*, $\Delta^{-1}(H_j^P) \le \Delta^{-1}(H_j^{P'})$. Hence, $\text{IHFLPOWA}(\langle u_1, H_1^P \rangle, \cdots, \langle u_n H_n^P \rangle)$

$$= \Delta^{-1}\left(\sum_{j=1}^{n} w_j \Delta^{-1}(H_j^P)\right) \le \Delta\left(\sum_{j=1}^{n} w_j \Delta^{-1}(H_j^{P'})\right) = \text{IHFLPOWA}(\langle u_1, H_1^{P'} \rangle, \cdots, \langle u_n, H_n^{P'} \rangle).$$

2) Boundary: According to the concept of union between two HFLTSs, we denote $env(H_i^P)=[H_{i^-}^P, H_{i^+}^P]$ for all i .Since $H_{i^-}^P \le s_{ki} \le H_{i^+}^P$ for all $k,i,$ we have $\min\{H_{i^-}^P | i=1,\cdots,n\} \le s_{ki} \le \max\{H_{i^+}^P | i=1,\cdots,n\}$ for all $k,i.$ Denote $s_\alpha = \min\{H_{i^-}^P | i=1,\cdots,n\}$ and $s_\beta = \max\{H_{i^+}^P | i=1,\cdots,n\}$, then $\Delta^{-1}(H_j^P) = \sum_{k=1}^{l_j} p_{kj}\Delta^{-1}(s_{kj}) \ge \sum_{k=1}^{l_j} p_{kj}\Delta^{-1}(s_\alpha) = \Delta^{-1}(s_\alpha)$, where $\Delta(\Delta^{-1}(H_j^P))$ is the aggregation result of the linguistic terms in H_i^P with the j th largest u_i, then we have

$$\sum_{j=1}^{n}\left(w_j\left(\sum_{k=1}^{l_j} p_{kj}\Delta^{-1}(s_{kj})\right)\right) = \sum_{j=1}^{n} w_j\Delta^{-1}(H_j^P) \ge \sum_{j=1}^{n} w_j\Delta^{-1}(s_\alpha) = \Delta^{-1}(s_\alpha). \text{ Similarly}$$

$$\sum_{j=1}^{n}\left(w_j\left(\sum_{k=1}^{l_j} p_{kj}\Delta^{-1}(s_{kj})\right)\right) \le \Delta^{-1}(s_\beta), \text{ i.e., } s_\alpha \le \Delta\left(\sum_{j=1}^{n}\left(w_j\left(\sum_{k=1}^{l_j} p_{kj}\Delta^{-1}(s_{kj})\right)\right)\right) \le s_\beta \text{ and}$$

$$\min\{H_{i^-}^P | i=1,\cdots,n\} \le IHFLPOWA \le \max\{H_{i^+}^P | i=1,\cdots,n\}.$$

3) Idempotency: If $env(H_i^P) = env(H^P)$ for all i and all of the H_i^P have the same probability information, *i.e.*, $P_i = P, \forall i,$ where P is the probabilistic weighting vector of H^P, then $H_i^P = H^P$ and $\Delta^{-1}(H_j^P) = \sum_{k=1}^{l_j} p_{kj}\Delta^{-1}(s_{kj}) = \Delta^{-1}(H^P)$ for all j , then

$$\text{IHFLPOWA}\left(\langle u_1, H_1^P\rangle, \cdots, \langle u_n, H_n^P\rangle\right) = \Delta\left(\sum_{j=1}^{n} w_j\Delta^{-1}(H^P)\right) = \Delta\left(\Delta^{-1}(H^P)\right).$$

Especially, if $s_{ki} = s_\alpha$ for all *i*,*k*, then $H_i^P = \{(s_\alpha, 1)\}$ and the IHFLPOWA operator is reduced to the ILOWA operator, *i.e.*,

$$\text{IHFLPOWA}\left(\langle u_1, H_1^P\rangle, \cdots, \langle u_n, H_n^P\rangle\right) = \text{ILOWA}\left(\langle u_1, s_\alpha\rangle, \cdots, \langle u_n, s_\alpha\rangle\right) = \Delta\left(\sum_{j=1}^{n} w_j\Delta^{-1}(s_\alpha)\right) = s_\alpha$$

By choosing different weighting vectors, the IHFLPOWA operator defined by (11) has different reductions, which are described as follows.

Property 1 In the IHFLPOWA operator, we have the following three reduced linguistic aggregation operators.

1. If $W = \left(\frac{1}{n}, \cdots, \frac{1}{n}\right)$, then the IHFLPOWA operator is reduced to the hesitant fuzzy linguistic arithmetic mean probability averaging (HFLMPA) operator, *i.e.*,

$$\text{HFLMPA}\left(\langle u_1, H_1^P\rangle, \cdots, \langle u_n, H_n^P\rangle\right) = \Delta\left(\sum_{j=1}^{n}\frac{1}{n}\Delta^{-1}(H_j^P)\right) = \Delta\left(\frac{1}{n}\sum_{i=1}^{n}\sum_{k=1}^{l_i} p_{ki}\Delta^{-1}(s_{ki})\right). \quad (12)$$

2. If P_i is a uniform distribution for all *i*, *i.e.*, $P_i = \left(\frac{1}{l_i}, \frac{1}{l_i}, \cdots, \frac{1}{l_i}\right)$ then the induced hesitant fuzzy linguistic probabilistic mean ordered weighted averaging (IHFLPMOWA) operator can

be obtained as follows:

$$\text{IHFLPMOWA}\left(\langle u_1, H_1^P\rangle, \cdots, \langle u_n, H_n^P\rangle\right) = \Delta\left(\sum_{j=1}^{n} w_j \Delta^{-1}\left(H_j^P\right)\right) = \Delta\left(\sum_{j=1}^{n}\left(w_j \sum_{k=1}^{l_j}\left(\frac{1}{l_j}\Delta^{-1}\left(s_{kj}\right)\right)\right)\right)$$

$$= \Delta\left(\sum_{j=1}^{n}\frac{w_j}{l_j}\left(\sum_{k=1}^{l_j}\Delta^{-1}\left(s_{kj}\right)\right)\right). \tag{13}$$

3. If $w_i = \dfrac{1}{n}$ and $p_{ki} = \dfrac{1}{l_i}$ for all k, i, then the IHFLPOWA operator will reduce to the hesitant fuzzy linguistic arithmetic mean probabilistic mean averaging (HFLMPMA) operator, *i.e.*,

$$\text{HFLMPMA}\left(\langle u_1, H_1^P\rangle, \cdots, \langle u_n, H_n^P\rangle\right) = \Delta\left(\sum_{j=1}^{n}\frac{1}{n}\Delta^{-1}\left(H_j^P\right)\right)$$

$$= \Delta\left(\sum_{i=1}^{n}\left(\frac{1}{n}\sum_{k=1}^{l_i}\left(\frac{1}{l_i}\Delta^{-1}(s_{ki})\right)\right)\right) = \Delta\left(\sum_{i=1}^{n}\frac{1}{nl_i}\left(\sum_{k=1}^{l_i}\Delta^{-1}(s_{ki})\right)\right). \tag{14}$$

Property 2 If $w_j = 1$ and $w_i = 0$ for $i \neq j$, then the IHFLPOWA operator defined by (11) reduces to the linguistic probabilistic aggregation (LPA) of H_i^P with the jth largest u_i, *i.e.*,

$$\text{IHFLPOWA}\left(\langle u_1, H_1^P\rangle, \cdots, \langle u_n, H_n^P\rangle\right) = \Delta\left(\sum_{j=1}^{n}\left(w_j \sum_{k=1}^{l_j}\left(p_{kj}\Delta^{-1}\left(s_{kj}\right)\right)\right)\right)$$

$$= \Delta\left(\Delta^{-1}\left(H_i^P\right)\right) = \Delta\left(\sum_{k=1}^{l_i}\left(p_{ki}\Delta^{-1}\left(s_{ki}\right)\right)\right). \tag{15}$$

Corollary 1 The *maximum-IHFLPOWA operator* is obtained if $\Delta^{-1}\left(H_i^P\right) = \max\left\{\Delta^{-1}\left(H_j^P\right) \middle| j = 1, \cdots, n\right\}$. The *minimum-IHFLPOWA operator* is obtained if $\Delta^{-1}\left(H_i^P\right) = \min\left\{\Delta^{-1}\left(H_j^P\right) \middle| j = 1, \cdots, n\right\}$.

Property 3 If there is only one linguistic term s_{α_i} in each hesitant fuzzy linguistic term set such that $p_{l_{(i)}i} = 1$, *i.e.*, $H_i^P = \left\{(s_{\alpha_1}, 0), \cdots, (s_{\alpha_i}, 1), \cdots, (s_{\alpha_{l_i}}, 0)\right\}$, then the IHFLPOWA operator defined by (11) is reduced to the induced linguistic OWA.

Based on Theorem 1 and Properties 1-3, we notice that there are many interesting advantages of the IHFLPOWA operator defined by (11), such as monotonicity, boundary and idempotency, furthermore, it is the extensions of many others linguistic aggregation operators (12)-(15). According to (11), we also notice that there are many disadvantages of the IHFLPOWA operator, such as there needs more information than others linguistic aggregation operators, i.e., weighting information, probability information and the order inducing variable are needed in the IHFLPOWA operator, naturally, its computation is more complex than others linguistic aggregation operators.

3.2 The Measures for Characterizing the Weighting Vector

The measures are used to characterize the weighting vectors of aggregation operators [49, 50], such as the degrees of orness and andness (also called as the attitudinal character), the entropy of dispersion, etc. According to Eq.(11), the measures of the IHFLPOWA operator are decided by the weighting vector and probability distribution of each hesitant fuzzy linguistic term set. Denote $s_\alpha = \min\{H_{i^-}^P | i=1,\cdots,n\}$ and $s_\beta = \max\{H_{i^+}^P | i=1,\cdots,n\}$. For convenience, we rewrite each hesitant fuzzy linguistic term set with probabilistic information as $H_i^P = \{(s_\alpha, p_{\alpha i}),\cdots,(s_\beta, p_{\beta i})\}$ such that $\sum_{l=\alpha}^{\beta} p_{l_i} = 1$ where $p_{l_i} = 0$ means $s_l \notin H_i^P$ and $s_l \in S' = \{s_\alpha, s_{\alpha+1},\cdots,s_\beta\}$, such as in Example 1,

$$H_1^P = \{(s_2,0.5),(s_3,0.3),(s_4,0.2),(s_5,0),(s_6,0),(s_7,0)\}.$$

Then the IHFLPOWA operator can be rewritten as follows:

$$\begin{aligned}\text{IHFLPOWA}\left(\langle u_1, H_1^P\rangle,\cdots,\langle u_n, H_n^P\rangle\right) &= \Delta\left(\sum_{j=1}^{n} w_j \Delta^{-1}\left(H_j^P\right)\right) \\ &= \Delta\left(\sum_{j=1}^{n} w_j\left(\sum_{l=\alpha}^{\beta} p_{lj}\Delta^{-1}(s_l)\right)\right) = \Delta\left(\sum_{l=\alpha}^{\beta}\left(\left(\sum_{j=1}^{n}\left(w_j p_{lj}\right)\right)\times l\right)\right).\end{aligned} \tag{16}$$

Denote $\omega_l = \sum_{j=1}^{n}\left(w_j p_{lj}\right)$, where p_{l_j} is the probability of s_l in H_i^P with the jth largest u_i, then the weight of s_l in the IHFLPOWA operator is ω_l and $\omega_l = 0, 0 \le l < \alpha, \beta < l \le g$ and IHFLPOWA operator is furthermore rewritten as follows:

$$\text{IHFLPOWA}\left(\langle u_1, H_1^P\rangle,\cdots,\langle u_n, H_n^P\rangle\right) = \Delta\left(\sum_{l=\alpha}^{\beta} \omega_l \times l\right) = \Delta\left(\sum_{l=\alpha}^{\beta} \omega_{\beta+\alpha-l} \times (\beta+\alpha-l)\right). \tag{17}$$

Based on (16) and (17), the degree of orness associated with the IHFLPOWA operator is calculated as follows:

$$\begin{aligned} orness(\text{IHFLPOWA}) &= \sum_{l=\alpha}^{\beta} \omega_{\beta+\alpha-l} \frac{(\beta-\alpha+1)-(l-\alpha+1)}{\beta-\alpha} \\ &= \sum_{l=\alpha}^{\beta} \omega_{\beta+\alpha-l} \frac{\beta-l}{\beta-\alpha}. \end{aligned} \tag{18}$$

Based on Yager's works [49], if $\omega_\alpha = 1$, then the IHFLPOWA operator is a pure "and" operator. If $\omega_\beta = 1$, then the IHFLPOWA operator is a pure "or" operator. The more close all the total weight is to ω_α, the closer the IHFLPOWA operator is a pure "and" operator. The more close all the total weight is to ω_β, the more close the IHFLPOWA operator is a pure "or" operator. Formally, the measure of "andness" is the complement of the "orness", for the IHFLPOWA operator, we have *andness* (IHFLPOWA)= 1-*orness* (IHFLPOWA). According to Eq. (18), *orness*(IHFLPOWA) has the following properties.

Property4 For any IHFLPOWA operator, if $w_j = 1$ and $w_i = 0 (i \ne j)$, then *orness*(IHFLPOWA) defined by (18) is reduced as orness for the linguistic probabilistic aggregation operator of H_i^P with the jth largest u_i, *i.e.*,

$$orness\ (\text{IHFLPOWA}) = \sum_{l=\alpha}^{\beta}\left(\sum_{j=1}^{n} w_j p_{\beta+\alpha-l,j}\right)\frac{\beta-l}{\beta-\alpha} = \sum_{l=\alpha}^{\beta} p_{\beta+\alpha-l,i}\frac{\beta-l}{\beta-\alpha}.$$

1. If $p_{l_i} = \frac{1}{\beta+\alpha-l}(l=\alpha,\alpha+1,\cdots,\beta)$, then

$$orness(\text{IHFLPOWA}) = \sum_{l=\alpha}^{\beta}\left(\frac{\beta-l}{\beta-\alpha}\times\frac{1}{\beta-\alpha+1}\right)=\frac{1}{2}.$$

In this case, the IHFLPOWA operator is reduced as the linguistic mean operator of H_j^P, *i.e.*,

$$\text{IHFLPOWA}\left(\langle u_1,H_1^P\rangle,\cdots,\langle u_n,H_n^P\rangle\right)=\Delta\left(\sum_{l=\alpha}^{\beta}\frac{l}{\beta-\alpha+1}\right)=\Delta\left(\frac{\alpha+\beta}{2}\right);$$

2. If $p_{\alpha_i}=1$, then $orness(\text{IHFLPOWA})=1\times\frac{\beta - \beta}{\beta - \alpha}=0$. in this case,the IHFLPOWA operator is reduced as the linguistic min operator of H_i^P with the *j*th largest u_i *i.e.*,

$$\text{IHFLPOWA}\left(\langle u_1,H_1^P\rangle,\cdots,\langle u_n,H_n^P\rangle\right)=\Delta(\min\{\alpha,\cdots,\beta\})=s_\alpha;$$

3. If $p_{\beta_i}=1$, then $orness(\text{IHFLPOWA})=1\times\frac{\beta - \alpha}{\beta - \alpha}=1$. in this case, the IHFLPOWA operator is reduced as the linguistic max operator of H_i^P with the *j*th largest u_i *i.e.*,

$$\text{IHFLPOWA}\left(\langle u_1,H_1^P\rangle,\langle u_2,H_2^P\rangle,\cdots,\langle u_n,H_n^P\rangle\right)=\Delta(\max\{\alpha,\cdots,\beta\})=s_\beta.$$

Intuitively, Property 4 explains that if $w_j=1$ and $w_i=0(i\neq j)$, then the bigger the value of *orness*(IHFLPOWA), the closer that the aggregation result is to the maximum; The smaller the value of *orness*(IHFLPOWA), the closer that the aggregation result is to the minimum.

Property5 If $\forall l,i,\ p_{li}=\frac{1}{\beta-\alpha+1}$, then $\omega_l=\sum_{j=1}^{n}w_j p_{lj}=\frac{1}{\beta-\alpha+1}\sum_{j=1}^{n}w_j=\frac{1}{\beta-\alpha+1}$ and

$$Orness(\text{IHFLPOWA})=\sum_{l=a}^{\beta}\omega_{\beta+\alpha-l}\frac{\beta-l}{\beta-\alpha}=\frac{1}{\beta-\alpha+1}\sum_{l=\alpha}^{\beta}\frac{\beta-l}{\beta-\alpha}=\frac{1}{2}.$$

Inspired by Yager's works [47], the entropy of dispersion for the IHFLPOWA operator can be represented as follows.

$$H(\text{IHFLPOWA})=-\sum_{l=\alpha}^{\beta}\omega_l\ln\omega_l. \tag{19}$$

To measure the divergence of the weights against the degree of orness-andness measure [51], the divergence of the weights for the IHFLPOWA operator can be represented as follows.

$$Div(\text{IHFLPOWA})=\sum_{l=\alpha}^{\beta}\omega_{\beta+\alpha-l}\left(\frac{\beta-l}{\beta-\alpha}-orness\left(IHFLPOWA\right)\right)^2 \tag{20}$$

To measure the degree of balance between favoring the higher-valued elements or lower-valued elements [52], we represent the balance of the IHFLPOWA operator as follows.

$$Bal(\text{IHFLPOWA})=\sum_{l=\alpha}^{\beta}\omega_{\beta+\alpha-l}\frac{\beta-\alpha+2-2(l-\alpha+1)}{\beta-\alpha}=\sum_{l=\alpha}^{\beta}\omega_{\beta+\alpha-l}\frac{\beta+\alpha-2l}{\beta-\alpha}. \tag{21}$$

According to (16) and (17), these measures have some particular cases as follows.

Property 6 For the IHFLPOWA operator, we have

1. If $w_j = 1, w_i = 0(i \neq j)$ and $p_{\beta_j} = 1$, *i.e.*, the probability p_{β_i} of s_β in H_i^P with the jth largest u_i is 1,then $\omega_\beta = 1$ and $\omega_l = 0(l \neq \beta)$, which means that the aggregated result is the maximum s_β, in which, *orness*(IHFLPOWA)=1, *H*(IHFLPOWA)=0,*Div*(IHFLPOWA)=0 and *Bal*(IHFLPOWA)=1.

2. If $w_j = 1, w_i = 0(i \neq j)$ and $p_{\alpha_j} = 1$, *i.e.*, the probability p_{α_i} of s_α in H_i^P with the jth largest u_i is 1,then $w_\alpha = 1$ and $w_\alpha = 0(l \neq \alpha)$, the aggregated result is the minmum s_α, in which, *orness*(IHFLPOWA)=0, *H*(IHFLPOWA)=0, *Div*(IHFLPOWA)=0 and *Bal*(IHFLPOWA)=-1.

3. If $W = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)$ and $\sum_{i=1}^{n} p_{li} = \frac{n}{\beta - \alpha + 1}$ for all $l(l \in \{\alpha, \alpha+1, \cdots, \beta\})$, then $\omega_l = \sum_{j=1}^{n} w_j p_{l_j} = \frac{1}{n} \times \frac{n}{\beta - \alpha + 1} = \frac{1}{\beta - \alpha + 1}$ for all *l*, in which, *orness*(IHFLPOWA)= $\frac{1}{2}$, $H(\text{IHFLPOWA}) = \ln(\beta - \alpha + 1)$, $Div(\text{IHFLPOWA}) = \frac{1}{6(\beta - \alpha)} + \frac{1}{12}$ and *Bal*(IHFLPOWA)=0.

4. Illustrative Case Study

In this section, as an example, we consider a movie fuzzy ontology creation based on the IHFLPOWA operator, which is initially described in [4]: A company, such as Film affinity, is interested in building a movie fuzzy ontology using the opinions of their users. In such a way, it can be consulted by users in order to find films that correspond to their interests. The company wants to classify 20 different movies (alternatives) $A = \{A_1, A_2, \cdots, A_{20}\}$ using the four following concepts $C = \{c_1, c_2, c_3, c_4\}$:

1. Action (c_1): Measures the amount of action in the film;
2. Humor (c_2): Takes into account if the film is comical;
3. Drama (c_3): Refers to whether the film storyline is sad and touching;
4. Mystery (c_4): Mystery films get high label values in this concept.

It should be noted that this is a brief movie fuzzy ontology example. Other concepts like argument, overall opinion, actors' performance, science fiction or horror could be added. Creating a functional movie fuzzy ontology is out of the scope of this article.

Because four concepts need to be measured using users' opinion, four different group decision making processes must be performed, such as for the action concept, users are asked to sort the films according to the level of action on them. Because a large amount of individuals are available, it is difficult for experts to carry out a pairwise comparison of all of them. In this example, a group decision making method that allows the participation of a large amount of experts and a high number of alternatives is followed. This method allows users to provide information only about the movies that they prefer. In such a way, a user chooses for themselves how many pairwise comparisons they wish to make. Because participation from a large number of users is expected, enough information to carry out a reliable group decision making process will be collected. Formally, in a movie fuzzy ontology $O_F = \{I, C, R, F, A\}$, assume individuals (movies) are $I = \{A_1, A_2, \cdots, A_n\}$, concepts associated with movies are $C = \{C_1, C_2, \cdots, C_m\}$ and users are $E = \{e_1, e_2, \cdots, e_p\}$, users provide their opinion about movies by using hesitant fuzzy linguistic term set with probabilistic information on a linguistic term set, then creating the movie fuzzy ontology based on the IHFLPOWA operator can be described as following steps:

Step 1. Let a linguistic term set be $S = \{s_0, s_1, \cdots, s_g\}$. User e_q provides his/her opinion matrix: $H^{P(q)} = (H_{ti}^{P(q)})_{n\times m}$, where $H_{ti}^{P(q)}$ indicates the hesitant fuzzy linguistic term set with probabilistic information provided by user e_q for the alternative A_t with respect to the concept C_i and each $H_{ti}^{P(q)} = \{s_{ti1}^{(q)}, s_{ti2}^{(q)}, \cdots, s_{til_{ti}}^{(q)}\}$, $l_{ti} = \left|H_{ti}^{P(q)}\right|$. Assume the weighting information about users are $V = (v_1, v_2, \cdots, v_p)$ satisfying that $\sum_{q=1}^{P} v_q = 1$ and $0 \le v_q \le 1$. Let the probabilistic weighting vector provided by the user e_q for the hesitant fuzzy linguistic term set $H_{ti}^{P(q)}$ be $H_{ti}^{P(q)} = \{p_{ti1}^{(q)}, p_{ti2}^{(q)}, \cdots, p_{til_{ti}}^{(q)}\}$ such that $\sum_{k=1}^{l_{ti}} p_{tik}^{(q)} = 1$ and $0 \le p_{tik}^{(q)} \le 1$.

Step 2. Assume the inducing variables $U = (u_1, u_2, \cdots, u_p)$. For the alternative A_t and concept C_i, we aggregate all $H_{ti}^{P(q)}(i = 1,2,\cdots,m)$ into an individual overall result by using the IHFLPOWA operator, i.e., the aggregation results of all users for alternative A_t with respect to C_i is a 2-tuple linguistic value $(s_t^{(q)}, \alpha_t^{(q)})$.

Step 3. Rank the n alternatives according to their results $(s_t^{(q)}, \alpha_t^{(q)})(t = 1,2,\cdots,n)$ with respect to each concept C_i, R and F are obtained and the fuzzy ontology is constructed.

This example focuses on preferences provided by a set of four users $E = \{e_1, e_2, e_3, e_4\}$. Each user provides his/her opinion about 4 movies according to the four concepts $C = \{c_1, c_2, c_3, c_4\}$ by using the hesitant fuzzy linguistic term sets on $S = \{s_0 = EL, s_1 = VL, s_2 = L, s_3 = SL, s_4 = M, s_5 = SH, s_6 = H, s_7 = VH, s_8 = EH\}$ with probabilistic information. The specific process can be described in the following:

Step 1. Assume that $V = (0.3, 0.2, 0.2, 0.3)$. Each user e_q provides its opinion $H_{ti}^{P(q)}$ for movie A_t concerning concept C_i in the form of hesitant fuzzy linguistic term sets on S, the first four movies of which are shown in **Table 1**, in which, $H^{P(1)}$ is evaluation linguistic matric of the first four movies $\{A_1, A_2, A_3, A_4\}$ concerning the four concepts $C = \{c_1, c_2, c_3, c_4\}$ provided by user e_1, such as $H_{34}^{P(1)} = \{s_6, s_7\}$ means that user e_1 evaluates movie A_3 with respect to concept C_4 by using the hesitant fuzzy linguistic term set $\{s_6, s_7\}$. Similarly, users e_2, e_3 and e_4 provide evaluation linguistic matrix $H^{P(2)}$, $H^{P(3)}$ and $H^{P(4)}$ for the first four movies $\{A_1, A_2, A_3, A_4\}$ concerning the four concepts $C = \{c_1, c_2, c_3, c_4\}$, respectively.

Table 1. Evaluation linguistic information of the first four movies provided by four users

		c_1	c_2	c_3	c_4
$H^{P(1)}$	A_1	$\{s_2, s_3\}$	$\{s_1, s_2, s_3\}$	$\{s_2\}$	$\{s_6, s_7\}$
	A_2	$\{s_3, s_4, s_5\}$	$\{s_5\}$	$\{s_2, s_3\}$	$\{s_3, s_4\}$
	A_3	$\{s_3, s_4, s_5\}$	$\{s_3\}$	$\{s_3\}$	$\{s_6, s_7\}$
	A_4	$\{s_3\}$	$\{s_4, s_5\}$	$\{s_5\}$	$\{s_7, s_8\}$
$H^{P(2)}$	A_1	$\{s_4, s_5, s_6\}$	$\{s_6, s_7\}$	$\{s_4, s_5, s_6\}$	$\{s_4, s_5, s_6\}$
	A_2	$\{s_6, s_7\}$	$\{s_7, s_8\}$	$\{s_6, s_7\}$	$\{s_3, s_4, s_5\}$
	A_3	$\{s_6\}$	$\{s_4, s_5\}$	$\{s_4, s_5\}$	$\{s_2, s_3\}$
	A_4	$\{s_5, s_6\}$	$\{s_4, s_5\}$	$\{s_3, s_4\}$	$\{s_5, s_6\}$
$H^{P(3)}$	A_1	$\{s_5\}$	$\{s_6, s_7, s_8\}$	$\{s_6, s_7\}$	$\{s_4, s_5, s_6\}$
	A_2	$\{s_1, s_2, s_3\}$	$\{s_4, s_5, s_6\}$	$\{s_4\}$	$\{s_3, s_4\}$
	A_3	$\{s_6, s_7\}$	$\{s_7\}$	$\{s_6, s_7\}$	$\{s_4\}$
	A_4	$\{s_7\}$	$\{s_6, s_7\}$	$\{s_4, s_5\}$	$\{s_5, s_6\}$
$H^{P(4)}$	A_1	$\{s_2, s_3\}$	$\{s_4\}$	$\{s_2, s_3, s_4\}$	$\{s_3\}$
	A_2	$\{s_2\}$	$\{s_2, s_3\}$	$\{s_3, s_4\}$	$\{s_1, s_2, s_3\}$
	A_3	$\{s_2\}$	$\{s_2, s_3, s_4\}$	$\{s_4\}$	$\{s_6, s_7\}$
	A_4	$\{s_2, s_3\}$	$\{s_3, s_4, s_5\}$	$\{s_4, s_5\}$	$\{s_7, s_8\}$

Furthermore, each user e_q provides its probabilistic vector $\left(p_{ti1}^{(q)}, p_{ti2}^{(q)}, \cdots, p_{til_{ti}}^{(q)}\right)$ for $P_{ti}^{P(q)}$ shown in **Table 2**, in which, $P_{ti}^{P(1)}$ is probabilistic information of evaluation linguistic terms of the first four movies $\{A_1, A_2, A_3, A_4\}$ concerning the four concepts $C = \{c_1, c_2, c_3, c_4\}$ provided by user e_1, such as $P_{34}^{P(1)} = (0.7, 0.3)$ means that probabilistic information of the hesitant fuzzy linguistic term set $\{s_6, s_7\}$ is $(0.7, 0.3)$, i.e., user e_1 provides $\{s_6, s_7\}$ to evaluate movie A_3 with respect to concept C_4 , more important, user e_1 prefers to use s_6 (with possibility 0.7) than s_7 (with possibility 0.3). Similarly, $P_{ti}^{P(2)}, P_{ti}^{P(3)}$ and $P_{ti}^{P(4)}$ are probabilistic information of evaluation linguistic terms of the first four movies $\{A_1, A_2, A_3, A_4\}$ concerning the four concepts $C = \{c_1, c_2, c_3, c_4\}$ provided by users e_2, e_3 and e_4, respectively.

Table 2. The probabilistic vectors of the first four movies provided by four users.

		c_1	c_2	c_3	c_4
$P_{ti}^{P(1)}$	A_1	(0.9, 0.1)	(0.2,0.3,0.5)	(1)	(0.4,0.6)
	A_2	(0.4,0.3,0.3)	(1)	(0.7,0.3)	(0.3,0.7)
	A_3	(0.4,0.4,0.2)	(1)	(1)	(0.7,0.3)
	A_4	(1)	(0.4,0.6)	(1)	(0.7,0.3)
$P_{ti}^{P(2)}$	A_1	(0.4,0.3,0.3)	(0.4,0.6)	(0.4,0.3,0.3)	(0.5,0.3,0.2)
	A_2	(0.6,0.4)	(0.3,0.7)	(0.6,0.4)	(0.2,0.6,0.2)
	A_3	(1)	(0.2,0.8)	(0.5,0.5)	(0.5,0.5)
	A_4	(0.7,0.3)	(0.4,0.6)	(0.5,0.5)	(0.8,0.2)
$P_{ti}^{P(3)}$	A_1	(1)	(0.3,0.3,0.4)	(0.6,0.4)	(0.3,0.5,0.2)
	A_2	(0.4,0.4,0.2)	(0.1,0.4,0.5)	(1)	(0.2,0.8)
	A_3	(0.5,0.5)	(1)	(0.8,0.2)	(1)
	A_4	(1)	(0.8,0.2)	(0.9, 0.1)	(0.4,0.6)
$P_{ti}^{P(4)}$	A_1	(0.5,0.5)	(1)	(0.5,0.3,0.2)	(1)
	A_2	(1)	(0.3,0.7)	(0.5,0.5)	(0.2,0.7,0.1)
	A_3	(1)	(0.2,0.2,0.6)	(1)	(0.6,0.4)
	A_4	(0.3,0.7)	(0.3,0.4,0.3)	(0.5,0.5)	(0.6,0.4)

Step 2. Assume $U=(9,8,4,6)$. Since $u_1>u_2>u_4>u_3$, thus HFLTSs with probabilistic information can be listed as $H_{t1}^{P(q)}>H_{t2}^{P(q)}>H_{t3}^{P(q)}>H_{t4}^{P(q)}$ for all *t*,*q*. For each movie A_t and concept c_i, the IHFLPOWA operator is used to aggregate the $H_{ti}^{P(q))}(i=1,2,3,4)$ into an individual overall value $(s_t^{(q)},\alpha_t^{(q)})$, such as for movie A_1 and concept c_1, we have

$$\text{IHFLPOWA}\left(\left\langle u_1,H_{11}^{P(1)}\right\rangle,\cdots,\left\langle u_4,H_{14}^{P(1)}\right\rangle\right)$$

$$=\Delta\big(0.3\times(0.9\times2+0.1\times3)+0.2\times(0.4\times4+0.3\times5+0.3\times6)+0.2\times(0.5\times2+0.5\times3)+0.3\times5\big)$$

$$=\Delta(3.61)=(s_4,-0.39),$$

that is, $(s_1^{(1)},\alpha_1^{(1)})=(s_4,-0.39)$, it also means that four users $E=\{e_1,e_2,e_3,e_4\}$ provide $(s_4,-0.39)$ to evaluate movie A_1 with respect to concept c_1. Similarly, we can obtain all evaluation results for each movie A_t with respect to concept c_i, which are shown in **Table 3**.

Table 3. Evaluation results of four movies provided by four users according to four concepts

	c_1	c_2	c_3	c_4
A_1	$(s_4,-0.39)$	$(s_3,0.43)$	$(s_5,-0.31)$	$(s_5,-0.01)$
A_2	$(s_5,-0.06)$	$(s_5,0.2)$	$(s_5,-0.36)$	$(s_3,0.25)$
A_3	$(s_4,0.34)$	$(s_4,-0.13)$	$(s_4,0.46)$	$(s_5,-0.13)$
A_4	$(s_4,0.17)$	$(s_5,0.05)$	$(s_4,0.33)$	$(s_6,0.39)$

Step 3. Rank movies. According to **Table 3**, we can rank the first four movies $\{A_1,A_2,A_3,A_4\}$ with respect to each concept c_i, such as for concept c_1, we obtain $A_2\succ A_3\succ A_4\succ A_1$. Similarly, we can obtain the ranking of movies with respect to each concept c_i, and the movie fuzzy ontology is constructed, i.e., all rankings of movies with respect to concepts, respectively. **Table 4** shows the first four movies fuzzy ontology, in which, for each concept c_i, the ranking of four movies is from top to down.

Table 4. The constructed movie fuzzy ontology of four movies based on IHFLPOWA operator

	R			
	c_1	c_2	c_3	c_4
The first four movies fuzzy ontology	A_2	A_2	A_1	A_4
	A_3	A_4	A_2	A_1
	A_4	A_3	A_3	A_3
	A_1	A_1	A_4	A_2

Theoretically, the movie fuzzy ontology can be constructed by using different linguistic aggregation operator, in this paper, we focus on the max-IHFLPOWA, min-IHFLPOWA, HFLMPA, IHFLPMOWA and HFLMPMA operators to create the movie fuzzy ontology, for example, we utilize the max-IHFLPOWA operator to aggregate $H_{1i}^{P(1)}(i=1,2,3,4)$ into an individual overall value $(s_1^{(1)},\alpha_1^{(1)})$ for the first four movie and concepts, i.e.,

$$\text{max-IHFLPOWA}\left(\left\langle u_1, H_{11}^{P(1)}\right\rangle, \cdots, \left\langle u_4, H_{14}^{P\langle 1\rangle}\right\rangle\right)$$
$$=\Delta\left(\max\left\{0.9\times2+0.1\times3, 0.4\times4+0.3\times5+0.3\times6, 0.5\times2+0.5\times3, 5\right\}\right)=\Delta\left(\max\left\{2.1, 4.9, 2.5, 5\right\}\right)$$
$$=\Delta(5)=s_5.$$

It means that evaluation result of movie A_1 with respect to concept c_1 provided by four users $E=\{e_1, e_2, e_3, e_4\}$ is s_5, after all evaluation results of movies with respect to concepts are calculated by the max-IHFLPOWA operator, we can construct the movie fuzzy ontology, which is based on the max-IHFLPOWA operator. The first four movies fuzzy ontology based on the max-IHFLPOWA operator and others linguistic aggregation operators are shown in **Table 5**, in which, “Max” means that the first four movies fuzzy ontology is constructed by using the max-IHFLPOWA operator, “IHFLPMOWA” means that the first four movies fuzzy ontology is constructed by using the IHFLPMOWA operator, “Min” means that the first four movies fuzzy ontology is constructed by using the min-IHFLPOWA operator, “HFLMPMA” means that the first four movies fuzzy ontology is constructed by using the HFLMPMA operator, “HFLMPA” means that the first four movies fuzzy ontology is constructed by using the HFLMPA operator and “IHLPOWA” means that the first four movies fuzzy ontology is constructed by using the IHLPOWA operator.

Table 5. The first four movies fuzzy ontologies based on six linguistic aggregation operators

Operat	Ontology				Operator	Ontology			
	c_1	c_2	c_3	c_4		c_1	c_2	c_3	c_4
Max	A_4	A_2	A_2	A_4	IHFLPMOWA	A_3	A_2	A_1	A_4
	A_3	A_1	A_3	A_1		A_2	A_4	A_2	A_1
	A_2	A_3	A_1	A_3		A_4	A_3	A_4	A_2
	A_1	A_4	A_4	A_2		A_1	A_1	A_3	A_3
Min	A_4	A_4	A_3	A_4	HFLMPMA	A_2	A_2	A_1	A_4
	A_1	A_3	A_4	A_2		A_3	A_4	A_2	A_3
	A_3	A_2	A_2	A_3		A_4	A_1	A_3	A_1
	A_2	A_1	A_1	A_1		A_1	A_3	A_4	A_2
HFLMPA	A_2	A_4	A_1	A_4	IHLPOWA	A_2	A_2	A_1	A_4
	A_3	A_2	A_2	A_1		A_3	A_4	A_2	A_1
	A_4	A_3	A_4	A_3		A_4	A_3	A_3	A_3
	A_1	A_1	A_3	A_2		A_1	A_1	A_4	A_2

In **Table 5**, we notice that six movies fuzzy ontologies are different each other, which are based on the max-IHFLPOWA, HFLMPA, IHFLPMOWA, HFLMPMA and IHFLPOWA operator, respectively. According to properties of linguistic aggregation operators, the first four movies fuzzy ontologies based on the max-IHFLPOWA operator and the min-IHFLPOWA operator maybe correspond with optimistic and pessimistic users' knowledge, respectively. In addition, the HFLMPA, IHFLPMOWA, HFLMPMA and IHFLPOWA operators take into account probabilistic information in HFLTSs, their movies fuzzy ontologies may be more acceptive than others operators in real world practices.

5. Conclusions

In this paper, we propose the IHFLPOWA operator to aggregate hesitant fuzzy linguistic terms with probability information, where each probability indicates the preference degree of linguistic term in the hesitant fuzzy linguistic term set. We analyze various interesting properties of the IHFLPOWA operator, which show that the IHFLPOWA operator is extensions of several existed linguistic aggregation operators. We design a method based on linguistic aggregation operators to create fuzzy ontologies when users utilize hesitant fuzzy linguistic terms to express their opinions with respect to concepts, an illustrative example of a movie fuzzy ontology is utilized to present the utility and efficiency of the IHFLPOWA operator, in addition, we compare with different linguistic aggregation operator to construct the movie fuzzy ontology.

Acknowledgements

This work has been partially supported by the National Natural Science Foundation of China (Grant No.61372187, 61702317, 61473239), the scientific and technological project of Sichuan Province (2016GZ0099), Development of Petroleum Equipment and Manufacturing Technology Science Foundation of Yan'an (Grant No. 2017WZZ-03-02) and D.S. Park's work is supported by MSIP (Ministry of Science, ICT and Future Planning), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2017-2014-0-00720) supervised by the IITP (Institute for Information & communications Technology Promotion) and the National Research Foundation of Korea (No. NRF-2017R1A2B1008421), and the Fundamental Research Funds for the Central Universities, China (GK201703059, GK201801004) as well as the Fund Program for the Scientific Activities of Selected Returned Overseas Professionals in Shaanxi Province (Grant No. 2017024).

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