Optimal Channel Sensing for Heterogeneous Cognitive Networks: An Analytical Approach

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Received September 2, 2013; revised October 27, 2013; accepted November 24, 2013; published December 27, 2013

Abstract

The problem of optimal channel sensing in heterogeneous cognitive networks is considered to maximize the system throughput performance. The characteristics of an optimal operating sensing point maximizing the overall system rate are investigated under several rate criteria including the sum rate, the minimum of the primary and secondary rates, and the secondary rate with a guaranteed primary rate. Under the sum rate criterion, it is shown that the loss by imperfect sensing is no greater than half of the sum rate achieved by the perfect time sharing approach in a two user case if the sensing point is optimally designed.

Keywords: Cognitive radio/networks, cross-layer design and methodoloies, optimization, channel sensing, receiver operating characteristics

This work was supported by the 2013 Yeungnam University Rearch Grant. This work was presented in part at the IEEE ICASSP Mar. 2008 [15].

http://dx.doi.org/10.3837/tiis.2013.12.002

1. Introduction

Heterogenous cognitive networks sharing the spectrum between two different wireless communication systems have gained much interest as a promising approach to mitigate the scarcity of available frequency bands [1]-[3]. The secondary subnet users with the low priority are allowed to access the same channel with the primary macro-network users with high priority in an opportunistic way. In a typical scenario, the opportunistic access of secondary subnet users initiates with channel sensing to find the opportunity of channel access. That is, subnet users sense the spectrum to determine its availability and access the channel depending on the sensing result. The performance of this initial channel sensing has an impact on the overall system throughput: the opportunity of a secondary subnet user to use the channel is lost in the case of a false alarm, whereas the primary macro-network user's transmission is collided when a secondary user miss-detects the primary transmission in the channel. Hence, system throughputs are important criteria for designing channel sensors in heterogeneous cognitive networks.

There have been various channel sensors for heteroneous cognitive networks. Many of these works focus on detection performance without considering system throughputs. Energy detection, matched filtering, and cyclostationary feature detection methods were introduced as three main categories of spectrum sensing [4]-[6]. A compressed sensing technique was proposed for wideband sensing [7], and joint multiband detection and collaborative sensing among multiple secondary users were examined in [8]-[11]. The design of spectrum sensing from the perspective of system throughputs has been studied in [12]-[15]. The joint design of sensing and access policy maximizing the throughput of a secondary user under a given miss-detection probability or equivalently primary rate was investigated in [12], [13]. In [14], the trade-off between sensing duration and throughput was examined with constraint on miss-detection probability. On the other hand, the optimal number of secondary users maximizing the total deliverable throughput through both primary and secondary networks was investigated in [16]. As in [16], the sum rate of primary and secondary networks will be a good performance measure in cognitive radio networks. In the approach guaranteeing the primary rate, the strict constraint on the primary user's data rate can cause harmful effects in maximizing spectrum utilization. If a primary user relaxes the strict constraint on the guaranteed rate, we can achieve the higher sum rate. While a primary network sacrifices a small rate loss, a secondary network can get large gain. This operation can be regarded as cooperation between two networks sharing the same spectrum with different priorities.

In this paper, we consider heteroneous cognitive networks in which a single primary macro-network user accesses its channel whenever it has a packet and a secondary subnet user accesses the same channel depending on the sensing outcome. In particular, characteristics of optimal channel sensing to maximize the overall system throughput (mainly focusing on the sum throughput) are investigated. The following characteristics of optimal sensing for the sum rate criterion are shown:

1) The optimal false alarm probability of channel sensing at the secondary transmitter increases monotonically with the activity of the primary user if the receiver operating characteristics (ROC) curve of the detector for channel sensing is concave.

2) A unique optimal operating point exists if the ROC curve is strictly concave.

3) The sum rate achieved at the optimal operating point is always less than or equal to that of the perfect time sharing between the primary and secondary users, but the loss in sum rate, which is caused by imperfect sensing, is no greater than half of the sum rate achieved by perfect time sharing.

2. System Model

We consider a heterogeneous cognitive network in which there are one primary transmitter-receiver pair and one secondary transmitter-receiver pair, as shown in **Fig. 1**. It is assumed that a transmission is slotted and synchronized with slot interval T. When a primary user periodically transmits packets including a pre-determined preamble, a secondary user can detect the starting point of the packet and sense the channel with the preamble of the primary packet. With this operation, the secondary user can synchronize with the primary network. At each slot the primary sender transmits a data packet to the primary receiver with probability $\gamma \in [0, 1]$, which is called the primary macro-network activity factor. Each transmission link is assumed to be a flat fading additive white Gaussian noise (AWGN) channel. The received signal at the primary receiver is given by

$$\begin{cases} H_1 : y_p[n] = h_p s[n] + w[n], & \text{if the primary TX transmits,} \\ H_0 : y_p[n] = w[n], & \text{otherwise,} \end{cases} \tag{1}$$

where $s[n]$ is the primary (complex) data symbols with unit average energy, h_p is the complex channel coefficient for the primary link, and $w[n]\sim CN(0, \sigma^2)$, i.e., $w[n]$ is zero-mean circularly symmetric complex Gaussian noise with variance σ^2. If the secondary transmitter is the sender that transmits at time n, the received signal for the secondary receiver is given by

$$y_s[n] = h_s s_s[n] + w_s[n], \tag{2}$$

where $s_s[n]$ and $w_s[n]\sim CN(0, \sigma^2)$ are the signal and noise for the secondary receiver, respectively, and h_s is the complex channel coefficient for the secondary link. For channel sensing at the secondary transmitter, we maintain a link between the primary transmitter and secondary transmitter. The received signal at the secondary transmitter is given by

$$\begin{cases} H_1 : y_{sen}[n] = h_{sen} s[n] + v_{sen}[n], & \text{if the primary TX transmits,} \\ H_0 : y_{sen}[n] = v_{sen}[n], & \text{otherwise,} \end{cases} \tag{3}$$

where h_{sen} is the complex channel coefficient between the primary transmitter and the secondary transmitter and $v_{sen}[n]\sim CN(0, \sigma^2)$.

The secondary sender employs a detector to sense the primary transmission. ROC is given by $\{(\alpha, \beta(\alpha))\}$ where α and $\beta(\alpha)$ are the false alarm probability and the detection probability, respectively, of the detector at the secondary sender. Channel sensing is performed during the initial T_s symbols for each slot (**Fig. 2**). If the channel is sensed to be idle, the secondary sender transmits a packet to its own receiver for the remaining $T-T_s$ symbols. Otherwise, it waits for the next time slot to sense the channel again. (Here, we assume that the secondary subnet transmitters always have packets to transmit.)

The data rates under perfect sensing ($\alpha = 0$ and $\beta(\alpha) = 1$) are given by [17]

$$C_p = \log\left(1 + \frac{|h_p|^2}{\sigma^2}\right) \tag{4}$$

and

$$C_s = \frac{(T - T_s)}{T}\log\left(1 + \frac{|h_s|^2}{\sigma^2}\right),\ i = 1,\ldots,N, \tag{5}$$

for the primary macro-network user and the secondary subnet user, respectively. We consider the sum rate of all users (both primary and secondary users). When the sensing is perfect, the sum rate is given by

$$R_{sum} = \gamma C_p + (1-\gamma)C_s \tag{6}$$

where γ is the primary macro-network activity factor. In practice, the sum rate given by (6) is decreased due to imperfect sensing. False alarm prevents the secondary sender from transmitting its data and miss-detection causes collision between packets. When such collision occurs, we assume that no transmission is successful. Incorporating the false alarm probability α and the detection probability $\beta(\alpha)$ into (6), the sum rates are rewritten as

$$R_{sum} = \gamma\beta(\alpha)C_p + (1-\gamma)(1-\alpha)C_s. \tag{7}$$

As a system model which we consider in this paper, the general cognitive radio with primary and secondary users can be assumed instead of a heterogeneous cognitive network. In general cognitive radio networks, however, the maximization of the secondary rate with a constraint on quality of service or rate of a primary link is considered. To justify the objective to maximize the sum rate, we employ the heterogeneous network.

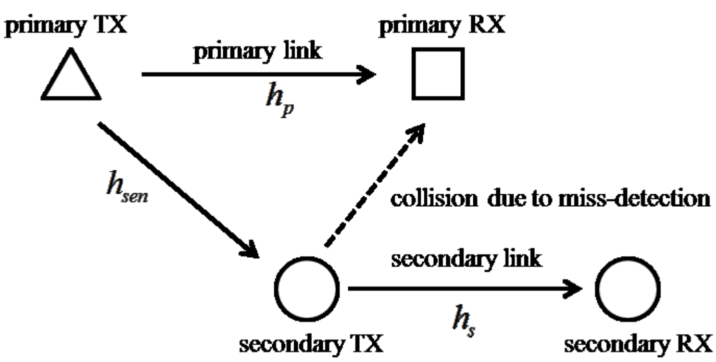

Fig. 1. Heterogeneous cognitive network model.

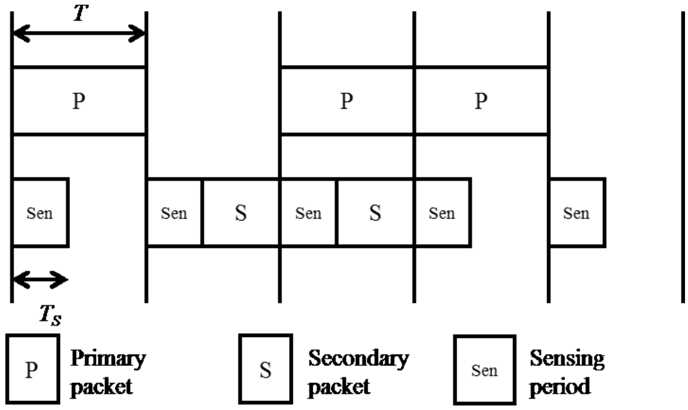

Fig. 2. System operation with sensing of secondary user.

3. Optimal Channel Sensing

In this paper, we consider three system rate criteria:
Case (i) a sum rate with a given primary activity factor,
Case (ii) the minimum of the primary and secondary rates,
Case (iii) the maximum of the secondary rate with a guaranteed primary rate.
While case (iii) is the most relevant to cognitive radio networks with a performance guarantee for the primary user, the other two criteria are also meaningful when we relax the strict performance guarantee for the primary user or when we consider the cooperation between the primary and secondary users to yield larger overall system throughput. In the latter case, the network can be viewed as being composed of multiple cooperating users with two priority classes.

3.1 Case Study: Matched Filter Detector

In most heterogeneous cognitive networks, a multi-stage sensing method, where a channel is roughly sensed with energy detection and then finely sensed with matched filtering, is used to improve sensing reliability. Because the performance of the multi-stage sensing method mainly depends on that of a matched filtering method, we analyze our results with a matched filtering scheme throughout this paper. A matched filter can be used if the initial T_s symbols of the primary signal are known. The ROC for a the matched filter case is given by

$$\beta(\alpha) = Q\left(Q^{-1}(\alpha) - \sqrt{\frac{2\,|h_{sen}|^2\, T_s}{\sigma^2}} \right) \tag{8}$$

$$= Q\left(Q^{-1}(\alpha) - \sqrt{2\mathrm{SNR}T_s} \right), \tag{9}$$

where $Q(\ )$ is the Gaussian tail probability and the secondary user sensing signal-to-noise ratio (SNR) is given by $|h_{sen}|^2/\sigma^2$ [18], [19]. The detection probability is always a monotonically increasing and concave function of the false alarm probability. **Fig. 3** shows the ROC of matched filtering with different sensing SNR values when $T_s = 100$.

Even in fading channels, it can be shown that the concavity of the average ROC is preserved. In Rayleigh fading channels, as an example, we can use the average ROC obtained with the exponential distribution of the sensing SNR, i.e.,

$$E\{\beta(\alpha)\} = \int_{g=0}^{\infty} \beta(\alpha)\frac{1}{\bar{g}}\exp\left(-\frac{g}{\bar{g}}\right)dg \tag{10}$$

where $\bar{g}$ is the mean of the exponential distribution. For any $\lambda \in [0, 1]$, we have

$$\begin{aligned} E\{\beta(\lambda\alpha_1+(1-\lambda)\alpha_2)\} &= \int_{g=0}^{\infty} \beta(\lambda\alpha_1+(1-\lambda)\alpha_2)\frac{1}{\bar{g}}\exp\left(-\frac{g}{\bar{g}}\right)dg \\ &\geq \int_{g=0}^{\infty} \{\lambda\beta(\alpha_1)+(1-\lambda)\beta(\alpha_2)\}\frac{1}{\bar{g}}\exp\left(-\frac{g}{\bar{g}}\right)dg \\ &= \lambda\int_{g=0}^{\infty} \beta(\alpha_1)\frac{1}{\bar{g}}\exp\left(-\frac{g}{\bar{g}}\right)dg+(1-\lambda)\int_{g=0}^{\infty} \beta(\alpha_2)\frac{1}{\bar{g}}\exp\left(-\frac{g}{\bar{g}}\right)dg \\ &= \lambda E\{\beta(\alpha_1)\}+(1-\lambda)E\{\beta(\alpha_2)\} \end{aligned} \tag{11}$$

Here, the inequality holds due to the concavity of the ROC. Therefore, the average ROC is also concave, i.e., the concavity of the ROC is preserved in fading channels. Therefore, the main results of this paper do not change even in fading channels.

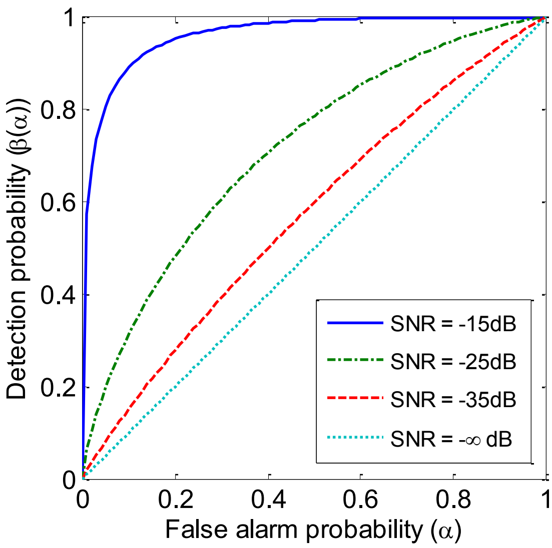

Fig. 3. The ROC of the matched filter with different sensing SNR when T_s=100.

The sum rate of heterogeneous cognitive networks as a function of α and γ is given by

$$R_{sum}(\alpha,\gamma) = \gamma\beta(\alpha)C_p + (1-\gamma)(1-\alpha)C_s. \tag{12}$$

For a given detector and sensing SNR, as well as a given link quality, we have one degree of freedom, i.e., choosing the false alarm rate α in our design. Hence, we can achieve the maximum sum rate by choosing an optimal operating point (α^{opt}, $\beta(\alpha^{opt})$) of the detector. When a matched filtering method is used for channel sensing, the optimal operating point can be obtained by solving the following equation:

$$Q'\left(Q^{-1}(\alpha^{opt})-\sqrt{2\mathrm{SNR}T_s}\right)Q^{-1}{}'(\alpha^{opt})=\left(\frac{1}{\gamma}-1\right)\frac{C_s}{C_p} \tag{13}$$

where $Q'(\cdot)$ and $Q^{-1}{}'(\cdot)$ denote the derivatives of $Q(\cdot)$ and $Q^{-1}(\cdot)$, respectively. It is hard to find the closed form optimal solution with (13). Since the ROC is concave for all types of detector, however, $R_{sum}(\alpha, \gamma)$ is the concave function of α. Using this concavity of the sum rate, we can find the optimal operating point with a gradient algorithm which is the well-known method to solve the convex optimization problem.

For example, **Fig. 4** shows the maximum sum rate and the optimal operating point with respect to the primary activity factor when C_p = 3.46 bps/Hz ($|h_p|^2/\sigma^2$ = 10) and C_s = 3.11 bps/Hz ($|h_s|^2/\sigma^2$ = 10, T = 1000, and T_s = 100) for a matched filter with -20dB sensing SNR. When γ = 0.7, the optimal operating point is (α^{opt} = 0.487, $\beta(\alpha^{opt})$ = 0.916) and the primary and secondary rates are given by 2.219 and 0.479, respectively. In this figure, γ = 1 corresponds to the extreme point in which the primary user always occupies the channel, and (α, $\beta(\alpha)$) = (1,1) is the optimal operating point maximizing the primary macro-network user's data rate. When γ = 0, on the other hand, the secondary transmitter can always use the channel and the (α, $\beta(\alpha)$) = (0, 0) is the optimal point. As the primary activity factor increases, the sum rate is affected depends on the primary link rather than the secondary link. Therefore, the operating point should be selected to increase the detection probability instead of reducing the false alarm. **Fig. 5** shows the maximum sum rate and the corresponding operating point with different primary capacities. As the primary capacity increases, it is important to reduce miss-detection events because the primary data rate in the sum rate becomes dominant. Thus, the optimal false alarm and corresponding detection probabilities should increase with the primary capacity.

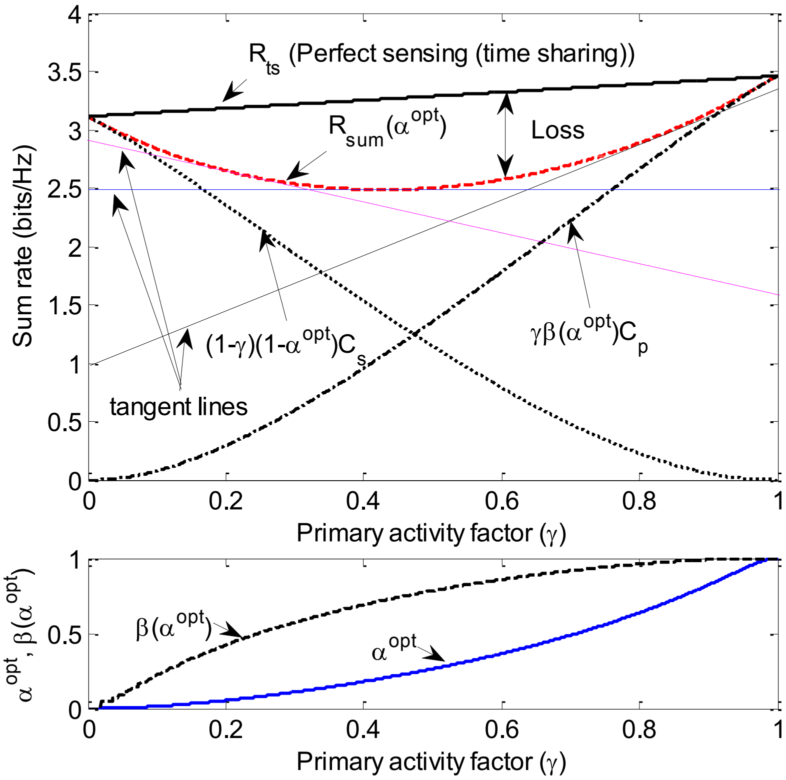

Fig. 4. R_{sum} and α^{opt} with different γ (C_p = 3.46 bps/Hz and C_s = 3.11 bps/Hz).

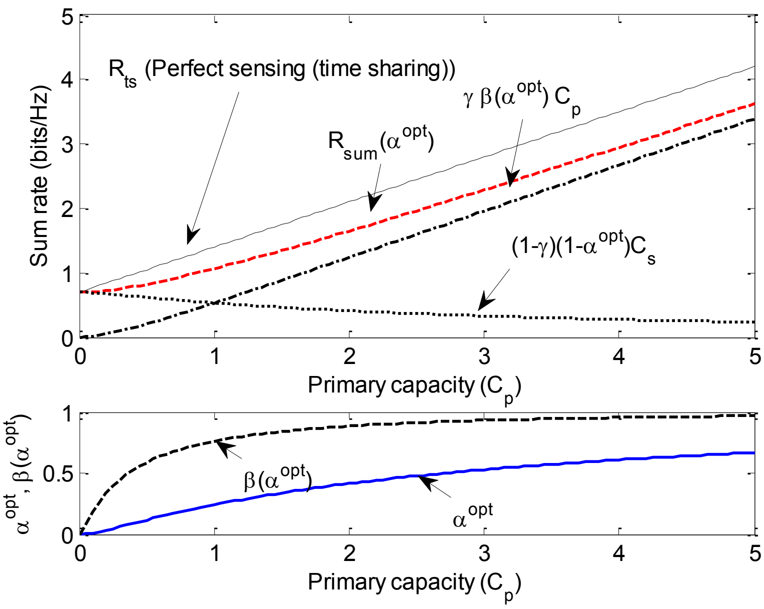

Fig. 5. R_{sum} and α^{opt} with different C_p (γ = 0.7 and C_s = 3.11 bps/Hz).

The maximum rate achieved with the optimal false alarm probability is a convex function of the primary activity factor as shown in **Fig. 4**. Using the time sharing method (with perfect sensing) between two extreme points at $\gamma = 0$ and $\gamma = 1$, on the other hand, we can achieve $R_{ts}(\gamma) = \gamma C_p + (1-\gamma)C_s$ as the sum rate. The convexity of $R^*(\gamma)$ implies that the sum rate with imperfect sensing is always less than that of perfect time sharing. To quantify this loss, we define the rate loss factor due to imperfect sensing as

$$L(\gamma) = \frac{R_{ts}(\gamma) - R^*(\gamma)}{R_{ts}(\gamma)}. \tag{14}$$

In the matched filtering case, the loss factor is written as

$$L(\gamma) = \frac{\gamma(1-\beta(\alpha^{opt}))C_p + (1-\gamma)\alpha^{opt} C_s}{\gamma C_p + (1-\gamma)C_s} \tag{15}$$

where $\beta(\alpha^{opt}) = Q\left(Q^{-1}(\alpha^{opt}) - \sqrt{2\mathrm{SNR}T_s}\right)$.

In the matched filtering case, the loss factor curves with respect to γ for different sensing SNR values are shown in **Fig. 6** when $C_p = 3.46$ bps/Hz and $C_s = 3.11$ bps/Hz. Additionally, the loss factor increases as the sensing SNR decreases. **Fig. 7** shows the loss factor as a function of sensing SNR for the matched filter and energy detector for the worst value of γ with the same parameters. It is seen that the loss factor converges to 1/2 from below as the sensing SNR decreases. As expected, the maximum loss factor of energy detection converges to 1/2 faster than that of the matched filter.

The necessary condition of the detector to show the results in this paper is the concavity of ROC. Though we use a matched filtering method to show the numerical results, the performance characteristics do not changed even with other types of detector.

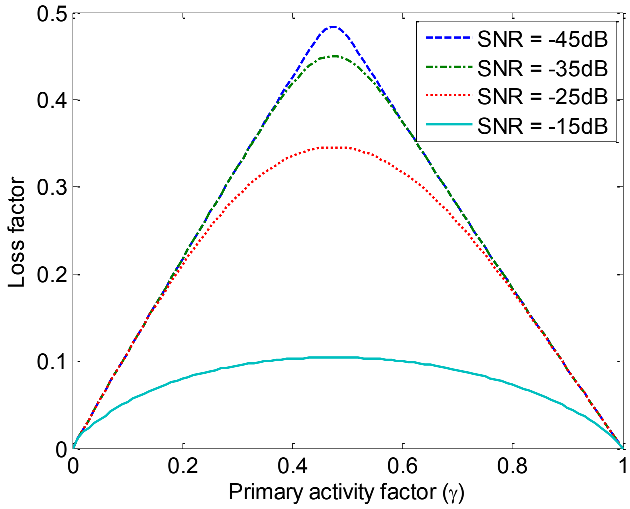

Fig. 6. Rate loss factor by using a matched filter for channel sensing ($C_p = 3.46$ bps/Hz and $C_s = 3.11$ bps/Hz).

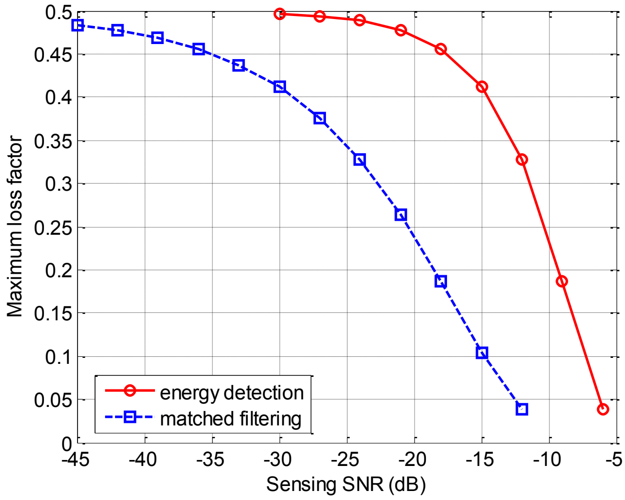

Fig. 7. Maximum rate loss factor of matched filtering and energy detection with a different sensing SNR (C_p = 3.46 bps/Hz and C_s = 3.11 bps/Hz).

3.2 Optimal Sensing Characteristics

The optimal operating point of the detector for the intermediate values of the primary activity factor is characterized in the following proposition.

Proposition 1: For any value of $\gamma \in (0,1)$, there exists an optimal operating $\alpha^{opt}(\gamma)$ when the ROC curve of the sensor is concave, i.e., $\beta(\alpha)$ is a concave function of α. Furthermore, $\alpha^{opt}(\gamma)$ is non-decreasing in this case as the primary activity factor γ increases. In the case of strict concavity, $\alpha^{opt}(\gamma)$ increases monotonically, and the optimal value is unique.
Proof: See Appendix.

The above proposition follows our intuition. When the primary user accesses the channel more actively, the secondary transmitter should allow more false alarms to reduce the miss-detection probability. When the channel is not frequently occupied by the primary user, the secondary transmitter should be more aggressive by reducing the false alarm rate. The proposition provides a sufficient condition for such intuition to be valid: *the ROC* (α, $\beta(\alpha)$) *is concave*. This property can be helpful to find the optimal operating point when the primary activity factor is periodically updated. If the primary activity factor increases, the new optimal operating point is higher than previous one. In this case, we can efficiently find the optimal solution by taking the search direction based on this property.

For each γ, we can optimize the sensing operating point and obtain the maximum sum rate given by

$$\begin{aligned} R^*(\gamma) &= \max_{\alpha} R_{sum}(\alpha,\gamma) \\ &= \gamma\beta(\alpha^{opt})C_p + (1-\gamma)(1-\alpha^{opt})C_s \end{aligned} \tag{16}$$

where α^{opt} is the solution to (13). The property of the optimal sum rate as a function of γ is summarized in the following proposition.

Proposition 2: The optimal sum rate $R^*(\gamma)$ (optimized over α for each γ) is a convex function of γ for any type of ROC curve.

Proof: See Appendix.

The convexity is clearly seen in the upper region in **Fig. 4**. $R^*(\gamma)$ is supported by all tangent straight lines determined by α, i.e., $R^*(\gamma)$ is a curve connecting the maximum point at each γ of all tangent lines associated with $\alpha \in [0, 1]$, as shown in the figure. Each tangent line is given by a linear function of γ, $\gamma\beta(\alpha)C_p$+$(1-\gamma)(1-\alpha)C_s$, with a given α. Also, α of each tangent line becomes optimal one α^{opt} at the point where the tangent line touches with the curve of $R^*(\gamma)$. The convexity of $R^*(\gamma)$ implies that the maximum $R^*(\gamma)$ occurs either at $\gamma=0$ or $\gamma=1$. That is, the maximum sum rate of the system is achieved when one user occupies the channel all the time under the collision model[1]. Using the time sharing method (with perfect sensing) between two extreme points at $\gamma=0$ and $\gamma=1$, we can achieve $R_{ts}(\gamma)=\gamma C_p+(1-\gamma)C_s$ as the sum rate. The convexity of $R^*(\gamma)$ also implies that the sum rate with imperfect sensing is always less than that of perfect time sharing. To quantify this loss, we define the rate loss factor due to imperfect sensing as

$$L(\gamma)=\frac{R_{ts}(\gamma)-R^*(\gamma)}{R_{ts}(\gamma)}. \tag{17}$$

One might expect this loss factor could be arbitrarily large, becoming 1 in the case of poor detection because of very low sensing SNR. However, this is not the case, and an upper bound for the loss factor exists and is given by the following proposition.

Proposition 3: With optimal design of false alarm rate α alone, the rate loss factor is no greater than 1/2 regardless of the value of γ and the sensing SNR, i.e.,

$$\max_{\gamma} L(\gamma) \leq 1/2. \tag{18}$$

Proof: See Appendix.

Fig. 6 shows the loss factor curves with respect to γ for different sensing SNR values when $C_p=3.46$ bps/Hz and $C_s=3.11$ bps/Hz. When the sensing SNR becomes -∞, $\beta(\alpha)=\alpha$ and the optimal operating point is given by

$$\lim_{\mathrm{SNR}\to-\infty}\alpha^{opt}=\begin{cases}1, & \text{if } \gamma C_p \geq (1-\gamma)C_s \\ 0, & \text{if } \gamma C_p < (1-\gamma)C_s\end{cases}. \tag{19}$$

When $\gamma C_p=(1-\gamma)C_s$ and the sensing SNR equals to -∞, the maximum rate loss factor equals to 1/2 regardless of the operating point. Additionally, the loss factor increases as the sensing SNR decreases. **Fig. 7** shows the loss factor as a function of sensing SNR for the matched filter and energy detector for the worst value of γ with the same parameters. It is seen, as predicted by Proposition 3, that the loss factor converges to 1/2 from below as the sensing

[1] This may not be valid under the interference model, where the transmit signal of one link is considered as interference of the other link, rather than the collision model.

SNR decreases. As expected, the maximum loss factor of energy detection converges to 1/2 faster than that of the matched filter.

When the sensing SNR is low, the sensing policy given by (19) guarantees the performance with small amount of loss. In **Fig. 7**, the additional loss due to employing the suboptimal operating point of (19) is lower than 10% when the matched filter is used for channel sensing and the sensing SNR is lower than -35dB.

3.3 Cases (ii) and (iii): Optimization under unknown Primary Activity and Primary Rate Guarantee

So far, we have assumed that the primary activity factor γ is known to the system and have focused on the sum rate. However, this may not be the case in practice. Suppose that the primary activity factor γ is unknown (i.e., the relative transmission activity between the primary and secondary users is unknown to the secondary transmitter) and the primary and secondary networks have equal priority. In such cases, the weighted sum rate may not be an appropriate criterion. One way to optimize the system rate in this case is to maximize the minimum rate of the two transmitter-receiver pairs, and the max-min criterion is given by

$$R_{max-min} = \max_{\alpha} \min \left\{ \beta(\alpha) C_p, (1-\alpha) C_s \right\}. \tag{20}$$

The optimal choice of sensing operating point α in this case is given by the following proposition.

Proposition 4: The optimal sensing operating point α_m^{opt} for the max-min criterion is given by the equalizer rule, i.e.,

$$\beta(\alpha_m^{opt}) C_p = (1-\alpha_m^{opt}) C_s. \tag{21}$$

Furthermore, this operating point corresponds to that of the primary activity factor γ, yielding the minimum $R^*(\gamma)$.

Proof: See Appendix.

Note that the sum rate optimal sensing point $\alpha^{opt}(\gamma)$ at the secondary transmitter requires the knowledge of the primary activity factor at the secondary transmitter. The max-min optimal sensing point α_m^{opt} can be used without the knowledge of γ. In this way, we can maximize the worst data rate between the two networks. Also note in **Fig. 4** that max-min point is the minimum point of $R^*(\gamma)$. On the left side of this point, the secondary user has priority while the primary user has priority on the right side of the point. Thus, the max-min sensing operation point corresponds to the sensing operation that equalizes the priorities of the primary and secondary transmitters. The max-min operating point is easily obtained from the sensing ROC, as shown in **Fig. 8**, by rewriting (21) as $\beta(\alpha_m^{opt}) = C_s/C_p(1-\alpha_m^{opt})$.

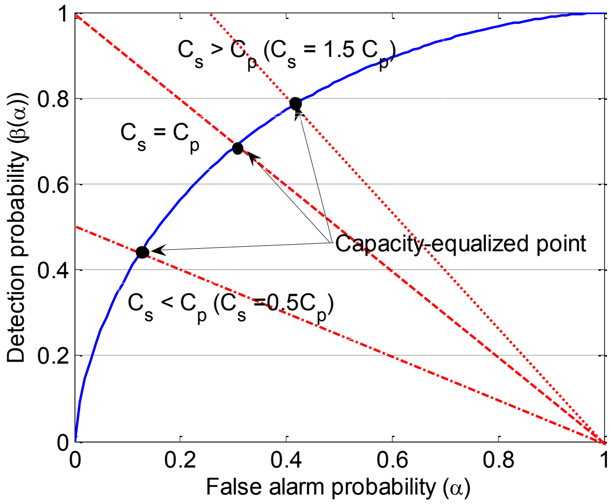

Fig. 8. Max-min solution for different C_s/C_p.

Now, consider the criterion maximizing the secondary rate while guaranteeing some target rate for the primary user. This approach is more relevant for cognitive radio networks where the priority of the primary user over secondary users is strict and the primary user does not cooperate with secondary users. Under the i.i.d. process model for the primary transmission, the problem is formulated as a constrained optimization:

$$\begin{aligned} &\max_{\alpha}(1-\alpha)C_s \\ &s.t.\ \ \beta(\alpha)C_p \geq R_{p,c} \end{aligned} \tag{22}$$

where $R_{p,c}$ is the minimum rate guaranteed for the primary user. In this case, the optimal sensing operating point $\alpha_{qos}{}^{opt}$ is simply given by

$$\beta(\alpha_{qos}^{opt})C_p = R_{p,c} \tag{23}$$

since the object function is a linear of α and $\beta(\alpha)$ is a monotonically increasing function of α. Therefore, $\alpha_{qos}{}^{opt}$ is on the boundary of the constraint.

4. Conclusions

We have considered the problem of optimal spectrum sensing in heterogeneous cognitive networks in which the primary network accesses the channel whenever it has a packet and the secondary network accesses the spectrum after sensing the spectrum. We have investigated the characteristics of the optimal operating sensing point that maximizes the system rate in such networks. We have shown that the loss by imperfect sensing is no greater than half of the sum rate of perfect time sharing. As a further work, we can consider the network with multiple primary and secondary users and investigate the characteristics of a sum rate and the optimal sensing under a given network condition.

Appendix

Proof of Proposition 1

The existence is straightforward from the continuity of R_{sum} as a function of α and the finite range of α. Hence, we prove the uniqueness and monotonicity. When R_{sum} is not a monotonic function of α, $\alpha^{opt}(\gamma)$ maximizing R_{sum} is given by solving the following equation:

$$\frac{\partial R_{sum}}{\partial \alpha} = \gamma \frac{\partial \beta(\alpha)}{\partial \alpha} C_p - (1-\gamma) C_s = 0. \quad \text{(A1)}$$

$$\left. \frac{\partial \beta(\alpha)}{\partial \alpha} \right|_{\alpha=\alpha^{opt}} = \left(\frac{1}{\gamma} - 1 \right) \frac{C_s}{C_p}. \quad \text{(A2)}$$

Due to the concavity of the ROC curve, $d\beta(\alpha)/d\alpha$ is a non-increasing function. The right-hand side of (A2) is also a monotonic decreasing function of γ. Therefore, $\alpha^{opt}(\gamma)$ is a non-decreasing function of γ. In the case of strict concavity, $d\beta(\alpha)/d\alpha$ is a monotone decreasing function of α, and the claim follows. In trivial cases where the sum rate is a monotonic increasing or decreasing function of α, the optimal false alarm is uniquely given by 1 or 0 regardless of γ, respectively. □

Proof of Proposition 2

Let $\gamma_\lambda = \lambda\gamma' + (1-\lambda)\gamma''$ $(0 \le \lambda \le 1)$ and $\bar{\gamma}_\lambda = \lambda\bar{\gamma}' + (1-\lambda)\bar{\gamma}''$, where $\bar{\gamma} = 1-\gamma$ and similarly for $\bar{\gamma}'$ and $\bar{\gamma}''$. Then, we have

$$\begin{aligned} R^*(\gamma_\lambda) &= \gamma_\lambda \beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + \bar{\gamma}_\lambda(1-\alpha^{opt}(\gamma_\lambda))C_s \\ &= (\lambda\gamma' + (1-\lambda)\gamma'')\beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + (\lambda\bar{\gamma}' + (1-\lambda)\bar{\gamma}'')(1-\alpha^{opt}(\gamma_\lambda))C_s \\ &= \lambda[\gamma'\beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + \bar{\gamma}'(1-\alpha^{opt}(\gamma_\lambda))C_s] + \\ &\quad (1-\lambda)[\gamma''\beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + \bar{\gamma}''(1-\alpha^{opt}(\gamma_\lambda))C_s] \\ &\le \lambda R^*(\gamma') + (1-\lambda)R^*(\gamma'') \end{aligned} \quad \text{(A3)}$$

The last step is by the definition of $R^*(\gamma)$. Here, conditions for the ROC curve are not required. □

Proof of Proposition 3

The optimal false alarm in non-trivial cases is determined by the (A2). As a SNR increases, α^{opt} decreases and $\beta(\alpha^{opt})$ increases due to (9). The $R^*(\alpha^{opt}, \gamma)$ is a monotonically decreasing function of a sensing SNR. Therefore, the loss factor is maximized when SNR approaches $-\infty$ and $\lim_{\mathrm{SNR} \to -\infty} \beta(\alpha) = \alpha$. When the sensing SNR approaches zero, we can rewrite the optimal sum rate as

$$\begin{aligned} \lim_{\mathrm{SNR} \to -\infty} R^*(\gamma) &= \lim_{\mathrm{SNR} \to -\infty} \max_\alpha R_{sum}(\alpha, \gamma) \\ &= \max_\alpha \gamma\alpha C_p + (1-\gamma)(1-\alpha)C_s. \end{aligned} \quad \text{(A4)}$$

Therefore, the optimal operating point is $\alpha^{opt} = 1$ when $\gamma C_p \ge (1-\gamma)C_s$. The maximum loss factor is given by

$$\max_{\gamma} L = \frac{(1-\gamma)C_s}{\gamma C_p + (1-\gamma)C_s} \le \frac{1}{2}. \tag{A5}$$

When $\gamma C_p < (1\text{-}\gamma)C_s$, the optimal operating point is $\alpha^{opt} = 0$ and the maximum loss factor is given by

$$\max_{\gamma} L = \frac{\gamma C_p}{\gamma C_p + (1-\gamma)C_s} < \frac{1}{2}. \tag{A6}$$

□

Proof of Proposition 4
Consider $R_{sum}(\gamma, \alpha)$. For a fixed α, it is a straight line as a function of γ. Hence, for a fixed α, the minimum value of R_{sum} over $0 \le \gamma \le 1$ occurs at either $\gamma = 0$ or $\gamma = 1$ with the minimum value of $\min\{\beta(\alpha)\, C_p, (1\text{-}\alpha)C_s\}$. Any straight line that is strictly below the $R^*(\gamma)$ curve does not achieve the max-min criterion since there is a tangent line to $R^*(\gamma)$ (and parallel to that line) that has a larger R_{sum}. The straight line tangent to $R^*(\gamma)$ at γ_0 is given by $R_{sum}(\gamma, \alpha^{opt}(\gamma_0))$. Since $R^*(\gamma)$ is convex due to Proposition 2 and $\min\{\beta(\alpha)\, C_p, (1\text{-}\alpha)C_s\}$ occurs at $\gamma = 0$ or 1 for straight line $R_{sum}(\gamma, \alpha^{opt}(\gamma_0))$, the max-min is achieved when $R_{sum}(\gamma, \alpha^{opt}(\gamma^*))$ touches $R^*(\gamma)$ and is parallel to the γ-axis, i.e., $\beta(\alpha_m^{opt})C_p = (1\text{-}\alpha_m^{opt})C_s$. (See **Fig. 4**.) γ^* is the primary activity factor yielding the worst weighted sum rate optimized over α. □

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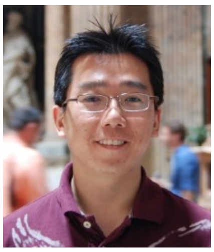

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