

# Resource Allocation in Multiuser Multi-Carrier Cognitive Radio Network via Game and Supermarket Game Theory: Survey, Tutorial, and Open Research Directions

**Omar B. Abdul-Ghafoor<sup>1</sup>, Mahamod Ismail<sup>1</sup>, Rosdiadee Nordin<sup>1</sup>, Musbah M. R. Shaat<sup>2</sup>**

<sup>1</sup>Department of Electrical, Electronic and System Engineering, University Kebangsaan Malaysia, Bangi, Malaysia

[e-mail: omar.omarghafoor@gmail.com], [mahamod@eng.ukm.my], [adee@eng.ukm.my]

<sup>2</sup> Centre Tecnològic de Telecomunicacions de Catalunya (CTTC), Barcelona - Spain

[e-mail: musbah.shaat@cttc.es]

\*Corresponding author: Omar B. Abdulghafoor

*Received April 30, 2014; revised July 8, 2014; accepted July 21, 2014; published November 30, 2014*

---

## Abstract

In this tutorial, we integrate the concept of cognitive radio technology into game theory and supermarket game theory to address the problem of resource allocation in multiuser multicarrier cognitive radio networks. In addition, multiuser multicarrier transmission technique is chosen as a candidate to study the resource allocation problem via game and supermarket game theory. This tutorial also includes various definitions, scenarios and examples related to (i) game theory (including both non-cooperative and cooperative games), (ii) supermarket game theory (including pricing, auction theory and oligopoly markets), and (iii) resource allocation in multicarrier techniques. Thus, interested readers can better understand the main tools that allow them to model the resource allocation problem in multicarrier networks via game and supermarket game theory.

In this tutorial article, we first review the most fundamental concepts and architectures of CRNs and subsequently introduce the concepts of game theory, supermarket game theory and common solution to game models such as the Nash equilibrium and the Nash bargaining solution. Finally, a list of related studies is highlighted and compared in this tutorial.

---

## 1. Introduction

Recent studies by the Federal Communication Commission (FCC) have shown that the conventional fixed spectrum allocation approach is becoming insufficient for addressing today's rapidly developing wireless communications, and there is a call for open spectrum access [1]. To meet this drastic demand in wireless spectrum, dynamic spectrum access (DSA) and cognitive radio (CR) were introduced as intelligent approaches/techniques to solve the problems associated with the fixed spectrum approach and have received significant interest from researchers (see [2] for a comprehensive review). Cognitive radio (also referred to as secondary user (SU) or unlicensed user) can be defined as a technique in which wireless devices have the ability to sense and discover a specific range in the frequency spectrum to identify currently unused bands (also called spectrum holes) for transmission purposes without interfering with the owner of the spectrum (also referred to as primary user (PU) or licensed user).

One of the most commonly occurring problems when addressing CRNs is allocation of the available resources (e.g., power and subcarrier) to CRs. This is because both PUs and CRs occupy the same spectrum band and transmit independently. Thus, CRs require flexible PHY to allow dynamic reconfiguration of the transmitted power and the signal frequency. One of the most promising candidates that can provide proper flexibility and high performance in CRN is the multicarrier technique [3].

Game theory is a mathematical tool that can be used to model scenarios in which the actions of decision makers, also called players, are in conflict. In CRNs, CR nodes attempt to access the licensed band, which belong to PUs, for their transmission purposes. Thus, the interaction among two decision makers (i.e., CRs and PUs) takes place, and game theory is shown to be an effective tool for analyzing and modeling resource allocation in such a scenario. The setup of spectrum allocation in CRNs is quite similar to the interaction of people in a real market, where the owner of the spectrum can 'rent' the temporary vacant band to CRs for their needs. This simple scenario makes adopting market theory as a game an interesting tool in modeling the problem of resource allocation in CRNs.

In this article, we have followed a simple strategy to introduce readers to the concept of (i) CRNs, (ii) game theory, (iii) supermarket game theory and (iv) resource allocation in Multiuser Multi-Carrier CRNs (MMC-CRNs) using game and supermarket game theory. Furthermore, we have tried our best to provide simple definitions and scenarios to facilitate a better understating of the problem of game/market resource allocation in MMC-CRNs.

### 1.1 Motivations for using game/supermarket game theory in MMC-CRNs

Game and supermarket game theory have become important fields of study for resource allocation in CRNs. This is because both "*game theory*" and "*supermarket game theory*" have proven to be a powerful decision making structure that able to provide excellent performance for CR nodes compared to that in ordinary optimization theory [4]. Moreover, game/supermarket game theory provides fast convergence of resource allocation algorithms

to a common point (i.e., steady-state point), which is another important issue to consider when adopting game/supermarket game theory in CRNs. The motivations behind adopting game and supermarket game theory in MMC-CRNs can be summarized as follows:

- (1) Interaction among independent users: In CRNs, there are two types of users (i.e., PUs and CRs), also called decision makers, with conflicts of interest, interacting with each other independently, trying to access the same spectrum band. This interaction adds certain obstacles in analyzing the problem of resource allocation in MMC-CRNs. Game theory, in contrast, appears to be one of the most attractive tools for removing these obstacles because it is mainly used to model scenarios where the action of one player impacts/conflicts with that of other players in the network [4]. Moreover, the availability of a common solution in game theory such as the Nash equilibrium and Nash bargaining add another advantage in modeling the problem of resource allocation via game theory.
- (2) Spectrum supermarket and real supermarket: The behavior of PUs and CRs in allocating their resources in MMC-CRNs is quite similar to the interactions among people in actual markets. Both include the following features: (i) *Pricing*, where the owner of the spectrum can gain benefits by renting the available spectrum holes to the tenants in the network (i.e., CR nodes). Thus, mutual benefits exist whereby the PUs improves their revenue and CRs enjoy access to the band for their needs. (ii) *Auction*, where CRs compete with each other in an auction scenario to obtain access to certain bands. The similarities between the concept of CRs and the interaction among people in a real supermarket make the adoption of economics concepts in analyzing the problem of resource allocation in MMC-CRNs another attractive tool.

## 1.2 Research Contributions

In this article, we have provided a tutorial on the application of game and supermarket game theory to the problem of resource allocation in MMC-CRNs. This tutorial is driven by the following problems: (i) how to build a cognitive radio network on a licensed spectrum; (ii) how CRs allocate their resources without harming the owner of the spectrum; (iii) how to apply game/market theory to the problem of resource allocation in MMC-CRNs; (iv) how to define the solution associated with game/market theory (e.g., Nash equilibrium, Nash bargaining), and (v) how to prove the existence and uniqueness in the defined game/market. We address these issues by making the following contributions:

- 1) Spectrum sharing and spectrum access in CRNs: We have provided a simple introduction to the concept of spectrum access and spectrum sharing techniques in CRNs. This includes related issues, objectives, and comparisons for recently developed spectrum sharing/access technique in CRNs.
- 2) Game theory and supermarket game theory: This includes the following:
  - We have provided a simple introduction and analysis to the fundamentals of (i) non-cooperative game theory, (ii) cooperative game theory, and (iii) market theory.

- Instead of mathematical approach for the NE, we have proposed a visual mathematical approach that facilitates a better understanding of the mathematical solution of the game (i.e., Nash equilibrium).
  - We have proposed several definitions, scenarios and examples for the application of game and market theory to the problem of resource allocation in MMC-CRNs.
- 3) Resource allocation and management in MMC-CRNs: We have included a survey of recent and related studies in the literature on the resource allocation problem in MMC-CRNs based on game/supermarket game theory by summarizing its main features and objectives.

To the best of our knowledge, this paper is the first tutorial that offers concrete descriptions related to the resource allocation problem in **MMC-CRNs** based on game and supermarket game theory.

### 1.3 Organization of the paper

This tutorial is organized as follows. In Section 2, we introduce the concept of (i) CR architecture, (ii) resource management in CRNs, and (iii) MMC-CRNs. The concepts of game theory and its solutions are presented in Section 3. Spectrum trading and supermarket game theory are explored in Section 4. An overview of resource allocation in MMC-CRNs and a survey of the related studies in the literature are presented in Section 5. Open research directions and the conclusion of the study are presented in Section 6 and Section 7, respectively.

## 2 Cognitive Radio Network

This section summarizes the main concept for (i) CRN models, (ii) spectrum sharing and spectrum access techniques in CRNs, and (iii) multicarrier techniques for CRNs.

### 2.1 CRN Models

In CRNs, there are two general types of models that can be defined as follows [5], [6]: (i) Infrastructure-based model: A CR base station, abbreviated as “CRBS”, is the main component in this approach and facilitates the residence of CRs in the licensed spectrum. Moreover, monitoring the spectrum band utilization and guiding the CRs to the vacant band is another feature of CRBS. (ii) Ad-hoc-based model: No permanent infrastructure exists in this approach. Thus, CRs must communicate among themselves independently to determine their actions while minimizing the amount of interference generated to PUs based on their own monitoring [6]. **Table 1** provides the main components for both approaches. In this tutorial, the application of game and supermarket game theory is considered in both infrastructure and ad-hoc approaches.

**Table 1.** Main Components of the CRN Models.

Architecture	Component
Infrastructure approach	<ol style="list-style-type: none"> <li>1. Primary user base station (PUBS): Coordinate licensed spectrum bands for PUs' usage.</li> <li>2. PUs: Ordinary mobile nodes, which are known as the owner of the spectrum, that have higher priority to use the available band.</li> <li>3. Cognitive radio base station (CRBS): Coordinate the coexistence of CRs and PUs in licensed spectrum.</li> <li>4. CRs/SUs: Cognitive radio users or secondary users that perform a spectrum sensing technique to utilize the vacant bands.</li> </ol>
Ad-hoc approach	<ol style="list-style-type: none"> <li>1. Primary user base stations (PUBS): As defined above.</li> <li>2. PUs: As defined above.</li> <li>3. CRs/SUs: Communicate in an ad-hoc fashion.</li> <li>4. CRBS: Not available in the ad-hoc approach.</li> </ol>

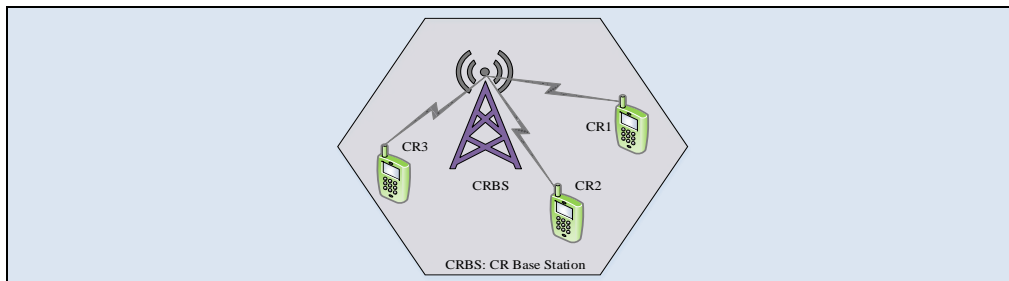
## 2.2 Spectrum Sharing and Recourse Management in CRNs

In this section, we summarize the features and concepts of spectrum sharing and spectrum access in CRNs.

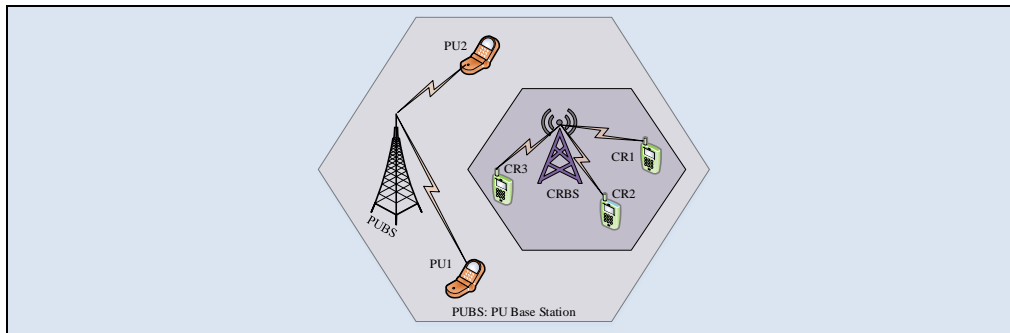
### 2.2.1 Spectrum Sharing in CRNs

A key challenge in CRN spectrum sharing is to answer the following question: “*how do you allocate transmission resources (e.g., power and subcarrier) efficiently among CRs over a wide range of available spectra with the available activities of neighboring PUs?*” [5] To be specific, spectrum sharing in CRNs must take into account the following essential issues: (i) providing the capability to maintain good QoS for CRs and (ii) minimizing the generated interference to the PUs by wisely assigning the transmission resources to CRs. To address the above mentioned issues, two main spectrum sharing techniques in CRNs are briefly described as follows [7]:

- 1) “*Open spectrum sharing*”: If CR users only access the unlicensed band, e.g., scientific and medical band (ISM band) or TV white space, then the spectrum sharing model is defined as an “*open spectrum sharing*”, where all CR users have the same rights among themselves to utilize the spectrum and allocate their resources accordingly. **Fig. 1** provides an example of open spectrum sharing.

**Fig. 1.** Example of open spectrum sharing in CRNs.

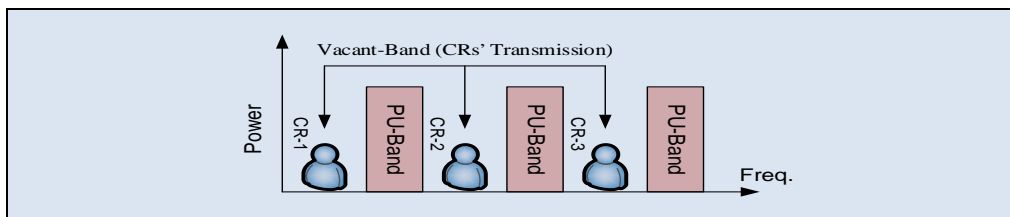
- 2) “*Hierarchical spectrum sharing*”: If CR users access the licensed spectrum together with PUs, then the spectrum sharing model is defined as a “*hierarchical spectrum sharing*”, where the CRs strictly follow certain policies (e.g., spectrum sensing) to access the available spectrum. **Fig. 2** provides an example of hierarchical spectrum sharing.



**Fig. 2.** Example of hierarchical spectrum sharing in CRNs.

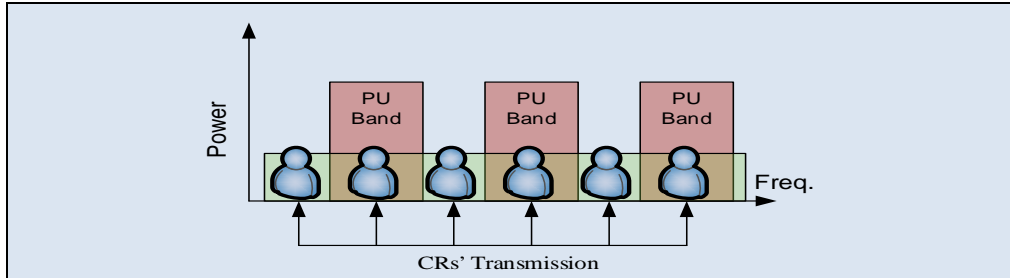
Moreover, hierarchical spectrum sharing can be divided into two main approaches: (i) overlay spectrum sharing and (ii) underlay spectrum sharing. The main features of the overlay and underlay approaches are summarized based on the following:

- 1) Overlay spectrum access: CRs, in this approach, have permission to access the available spectrum opportunistically/rationally if and only if the spectrum is not being occupied by the PUs. Moreover, CRs are able to use a portion of their power for cognitive transmission and the rest of the power to help (relay) the transmission of PUs [8]. The main objective of this technique is to manage and control the access of CRs to spectrum holes [5], [9]. **Fig. 3** shows an example of overlay spectrum access.



**Fig. 3.** Overlay spectrum access.

- 2) Underlay spectrum access: CRs, in this approach, share the spectrum simultaneously with PUs. Thus, PUs must be protected by applying a spectral mask on the CRs signals to make the generated interference from active CRs below the acceptable threshold for PUs to run smoothly and to provide fair communication services among PUs and CRs [8]. **Fig. 4** illustrates an example of underlay spectrum access.



**Fig. 4.** Underlay spectrum access.

Both spectrum access techniques have been widely adopted in the literature to allow CRs to communicate among them using the licensed spectrum band. However, more attention is being given to analyzing the underlay spectrum access caused by the difficulties associated with controlling the behavior of CRs attempting to minimize the generated interference to the owner of the spectrum.

### 2.3 Multiuser Multicarrier CRNs (MMC-CRNs): Concepts and Interference Analysis

In CRNs, the key features of CRs are (i) their ability to sense the available spectrum band and (ii) their ability to communicate with each other without interfering with the service of PUs to obtain better spectrum utilization than that in the fixed spectrum approach [10]. To fulfill the first point<sup>1</sup>, CRs must be prepared with spectrum sensing capability, and to achieve the second point, the PHY of the CR must be sufficiently flexible.

The multicarrier technique, in contrast, is envisioned as a promising candidate for CRNs that can satisfy the PHY issues for CRs. Moreover, multicarrier technique, abbreviated as (MC) have been seen to provide reliability and flexibility in allocating the available resources among CRs, which results in a better communication environment among CRs and PUs [11-13]. Additionally, MC-based CRNs can provide excellent coexistence among PUs and CRs based on the following abilities: (i) nulling the subcarriers that are currently occupied by active PUs and (ii) nulling the subcarriers that may produce certain amounts of interference for other users in the network [14].

Orthogonal frequency division multiplexing (OFDM) is a special case of MC techniques and is considered one of the most promising multicarrier candidates that can provide proper performance and flexibility in dynamically allocating spectrum holes among CRs. Moreover, adopting OFDM to the problem of resource allocation in CRNs facilitates the monitoring of the PU's activity and the occupancy of spectrum holes accordingly [3]. Another multicarrier candidate that can be adopted in the problem of resource allocation is the filter bank multicarrier (FBMC<sup>2</sup>) technique [15]. Compared to OFDM, (i) FBMC can provide better

<sup>1</sup> The first point (i.e., spectrum sensing technique) is beyond the scope of this tutorial article.

<sup>2</sup> Please refer to [16] for a comprehensive tutorial on OFDM and FBMC.

spectral efficiency in CRNs by separating the transmission of PUs and CRs through filtering [16], and (ii) FBMC promises very low out-of-band energy for each subcarrier signal [17]. Knowing this, in multiuser multicarrier networks, a multiple access scheme is required, e.g., an orthogonal frequency multiple access technique (OFDMA), to allocate both subcarriers and power to CR nodes.

To demonstrate the idea of MMC-CRN and the related issues, we provide the following motivating example

*Motivating Example\_1: “MMC-CRN Architecture and Interference Analysis”:* Assuming that we have a CR-based-OFDM network, consisting of two types of mobile radio devices (PUs and CRs) coexisting in the same geographical area and communicating using the same band as shown in Fig. 5-a, where communication links and channel gains of different links can be defined as follows: (i) the solid lines indicate the intended signal links; (ii) the spotted lines are the interference links; (iii)  $g_i^c$  and  $g_k^p$  are the interference gains from CR-to-PU and PU-to-CR, respectively; and (iv) the superscripts (*c*) and (*p*) refer to the cognitive radio and the primary users, respectively.

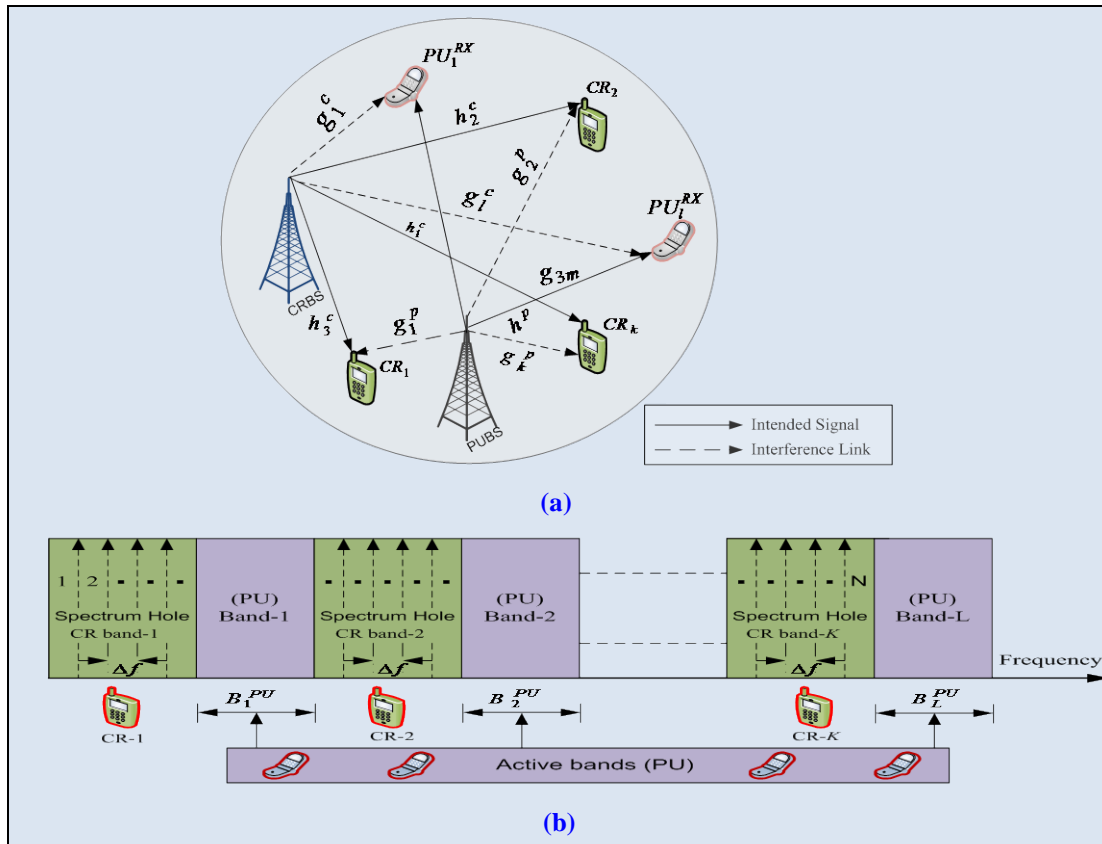


Fig. 5. a) Conceptual interference model in CRNs. b) Frequency distribution of PU activities.



Moreover, in MMC-based CRNs (i.e., OFDM), both CRs and PUs exist in side-by-side spectrum bands [18] as shown in Fig. 5-b. Thus, a mutual interference among PUs and CRs arises in this scenario and requires special consideration to maintain acceptable performance in both networks [19]. Assuming that CRNs consist of  $K$  CRs and the available band is divided into  $N$  subcarrier with  $\Delta f$  bandwidth, based on Fig. 5-(a & b), the generated interference from CRs to the band of active PUs can be defined based on the following definition:

**Definition 1:** [19-21]: Interference generated by the  $k$ -th CR access on the  $n$ -th subcarrier to the  $l$ -th PU denoted by  $I_{n,l}^{PU}$  can be defined as the integration of the power density spectrum of the  $n$ -th subcarrier ( $\phi_n$ ) across the  $l$ -th PU band (Band- $L$ ). Mathematically speaking, the interference introduced by the CR's signal can be modeled as

$$I_{n,l}^{PU} = p_n^k g_l^c \int_{d_l - B_l/2}^{d_l + B_l/2} \phi_n(f) df \quad (1)$$

where  $d_l$  is the distance between the  $n$ -th subcarrier and the  $l$ -th PU's spectrum band,  $p_n^k$  is the transmitted power for the active CR,  $g_l^c$  is the channel gain of the interference link from CR-to-PU and  $B_l$  is the  $l$ -th PU' bandwidth.

Note that the active PUs also generates an amount of interference to the CRs, and that amount of interference should be formulated mathematically<sup>3</sup> to provide for a concrete analysis for mutual interference in MMC-CRNs.

### 3. Application of Game Theory in MMC-CRNs

Details related to the concepts and applications of game theory including both “non-cooperative game theory” and “cooperative game theory” are discussed in the following sections.

#### 3.1 Game theory: Basic Concepts

Game theory was first introduced by J.V. Neumann and O. Morgenstern in 1944 [22] and is extensively used in microeconomics. Its application has commonly been recognized as a great tool for analyzing several engineering problems. Game theory can be defined based on definition 2.

<sup>3</sup> Please refer to appendix-I for details on interference' mathematical formulation.

**Definition 2:** "Game theory" [23-25]: Game theory is a branch of applied mathematics that can be adopted to study situations where the actions of several decision makers (also called players) are in conflict. Mathematically speaking, a game can be defined as [22], [25]

$$G_{ame} = \langle K, A, \pi_i \rangle, \quad (2)$$

where

- 1)  $K$  is a finite set of decision makers (players);
- 2)  $A$  is the Cartesian product of the sets of actions available to each player; and
- 3)  $\pi_i$  is the utility/payoff/objective functions of player  $i$ , which is a function of the action chosen by player  $i$  ( $a_i^{cton}$ ) and the actions chosen by all of the players in the game but not that of player  $i$  ( $a_{-i}^{cton}$ ).

Moreover, game theory can be classified into two main approaches: (i) Non-cooperative game theory: In this approach, the decision makers (or players) behave selfishly, aiming to maximize their own revenue. (ii) Cooperative game theory: In this approach, the players behave cooperatively to maximize the revenue of their network.

Furthermore, the strategies in game theory can be divided into two types: (i) pure strategy and (ii) mixed strategy. **Table 2** gives a brief comparison among players' strategies in game theory.

**Table 2.** Comparison of player behavior in a non-cooperative game<sup>4</sup>.

Game Model	Description	Type of Equilibrium
Pure strategy game	A normal or strategic game is considered the basic model in game theory. The players, in this case, are assumed to carry out only deterministic strategies (called pure	Nash equilibrium (NE)/correlated equilibrium (CE)
Mixed strategy game	In a mixed strategy game, the actions of the players are described by probability distributions.	Mixed strategy Nash equilibrium

In addition, it is worth mentioning that in [26] and [27], the authors provide a comprehensive survey on the application of game theory in CRNs and general wireless networks respectively. Furthermore, details related to fundamentals and concepts of game theory have been included as well. Hence, readers are advised to refer to [26] and [27] for more details regarding the mathematical formulations that illustrate the concepts of game theory. Our work, in contrast focuses on the applications of game theory and supermarket game theory to the problem of resource allocation in MMC-CRNs.

To simplify the concept of game theory and to show how the components of game theory and the elements of MMC-CRNs related to each others, we provide the following scenario as follows

<sup>4</sup> Please refer to [26] for more details on the concept of pure strategy game, mixed strategy game and correlated equilibrium (CE)

**Motivating Scenario\_1:** assuming that we have an OFDM-based CRNs coexisting with primary user provider in the same geographical area (i.e., similar to the deployment illustrated in Fig.5.a and 5.b). The CRNs consisting from three users competing with each other in order to allocate the subcarriers left over by the primary user. Game theory, in contrast, shows to be one of the most attractive tools to analyze this scenario because of the availability of group of users competing with each other to win a band for their transmission purposes. Hence, the component of game theory can be translated to the entities of MMC-CRNs according to Table.3

**Table 3.** Link between Game' component and the elements of MMC-CRNs

Basic Component of Game	Equivalent MMC-CRN' Elements
Players	Players are the cognitive nodes available in the network
Strategy Space	Strategy space, according to scenario_1, is the subcarriers left over by the PUs.
Utility Function	The utility function depends on the network setting and it may be maximization of throughput or minimization of some cost function in order to mitigate the generated interference to the owner of the spectrum.

In the following sections, details of both branches of game theory are presented by providing definitions, examples, scenarios and discussion regarding the common solution when a game is adopted. Hence, the interested readers can better understand the concept of game theory and its applications in the problem of resource allocation in MMC-CRNs.

### 3.2 Non-cooperative Game Theory Approach: Concepts and Theorems

Non-cooperative game theory is widely adopted in modeling resource allocation problem in CRNs and can be defined based on *definition 3*:

**Definition 3:** [23], [24]: Non-cooperative game theory, abbreviated as *NCGT*, is an approach that can be adopted to resolve the conflicts in a given scenario where the players are selfish and make their decisions independently.

The motivation of adopting *NCGT* in the problem of resource allocation in MMC-CRNs is the noticeable improvement in term of efficiency, spectrum utilization and the ability to guide selfish players to more stable resource allocation outcomes. To familiarize the reader with the concept of *NCGT*, we provide the following scenario:

**Scenario 2 “Application of NCGT in MMC-CRNs”:** NCGT is considered to be one of the most powerful tools used to investigate the problem of resource allocation in MMC-CRNs. In this scenario, the rational players, also called CRs, compete with each other to access vacant subcarriers. Thus, NCGT is the preferred approach to explore this scenario because it is generally applied to study situations in which the players’ objectives are in conflict. The game in this scenario can be modeled as follows:

- 1) Players  $K = \{1, 2, \dots, K\}$ : Players in the MC-CRN are CR nodes.
- 2) Players’ strategy  $A = A_1 \times A_2 \times \dots \times A_N$ : The set of strategies available to tenants. This includes subcarriers, bit loading and power allocation based on the local information available to players [28].
- 3) Utility function  $\pi_i : A \rightarrow R$  is the objective function that each player wishes to maximize (e.g., the player’s rate maximization) [29].

### 3.2.1 Common Solution to NCGT

When using NCGT, one should answer the following question: “*What will occur when interactions among rational players take place in certain applications?*” One of the most commonly used solutions to predict the output of a game is the *Nash equilibrium*, which can be defined based on *definition 4*.

**Definition 4:** [23, 24], [30]: The Nash equilibrium (termed NE) of a NCGT is an action profile such that no player can gain any benefit by varying his own strategy unilaterally.

To simplify the idea of the NE, we provide a visual approach to describe how the NE works in a given scenario as presented in **Fig. 6**.

Another significant issue in the solution of NCGT (or NE) is the investigation of two important properties: (i) Existence of an NE: The existence of an NE can be obtained using specific mathematical properties related to certain utility functions (e.g., supermodularity and supermodular games) [31]. (ii) Uniqueness: In addition to the existence property, the uniqueness of an NE must also be considered in the solution to NCGT. Moreover, **Theorem 1** and **Theorem 2** provide the necessary conditions for both properties as follows:

**Theorem 1** [29], [32]: An NE exists in a game  $G_{ame} = \langle K, A, \{\pi_i\} \rangle$  if for  $\forall i \in K$ , the following conditions hold:

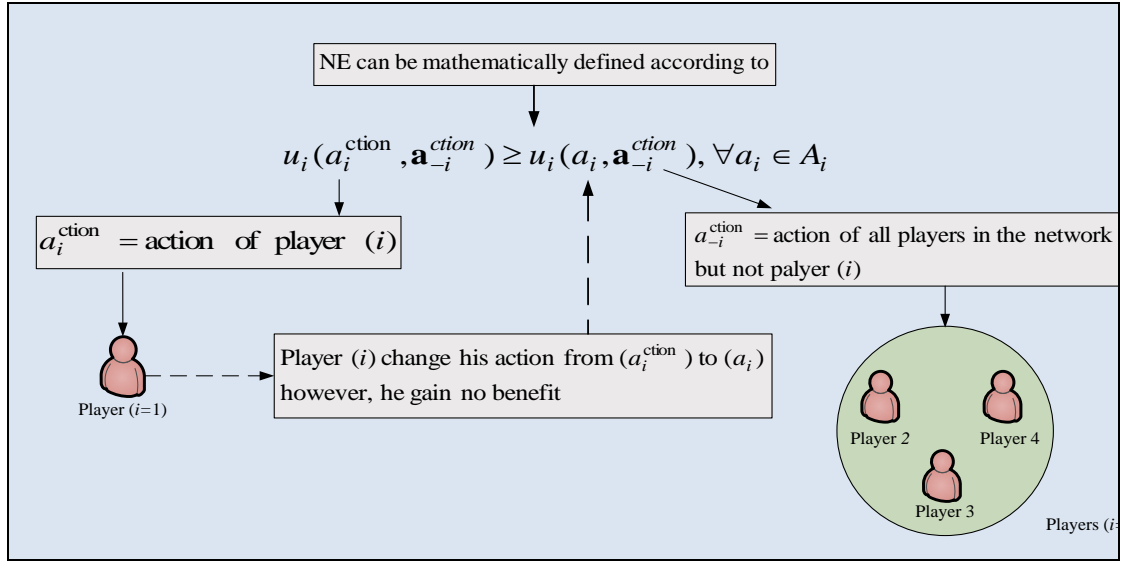
- 1) The action profile of player  $i$  is a nonempty, convex and compact subset of some Euclidean space.
- 2) The utility function  $\pi_i$  is a continuous and quasi-concave function over its action set.

**Theorem 2:** An NE in NCGT is unique if a game  $G_{ame} = \langle K, A, \{\pi_i\} \rangle$  modeled using the following special game technique is thus shown to reach a unique NE:

- 1) Potential game [33]<sup>5</sup>;
- 2) Standard function [34]<sup>6</sup>.

<sup>5</sup> Please refer [33] for additional details about potential game.

If a utility function in NCGT is carefully selected and the above mentioned theorems are fulfilled, then the NE is guaranteed to exist.



**Fig. 6.** Visual description for the NE.

### 3.3 Cooperative Game Theory Approach: Concepts and Theorems

In contrast to NCGT, players in cooperative game theory (abbreviated as CGT) are collaborating with each other wisely to maximize the total utility of their network, and, thus, the performance of the network can be improved accordingly. In this section, we discuss two popular forms of cooperative game theory: (i) bargaining game and (ii) coalition game.

#### 3.3.1 Bargaining Game

In bargaining game, abbreviated as BG, the players have a choice to cooperate and negotiate with each other. Thus, the players have the opportunity to reach a commonly beneficial agreement where all the players gain the maximum profit [35], [36]. The idea of a bargain game can be explained via the following scenario:

**Scenario 3 “Application of CGT (BG) in MMC-CRNs”:** BG is a powerful tool that provides fairness among players in the problem of resource allocation in MMC-CRNs. Players (i.e., CRs) cooperate with each other without any competition to maximize the revenue of their network. To simplify this scenario, we consider a two-player BG that negotiates to allocate subcarrier in OFDMA-CRN. Thus, the BG in this scenario can be described as follows:

- 1) Conflicts and fairness issue: Free sub-carriers are available to both players. However,

<sup>6</sup> Please refer to [34] for additional details about the standard function and uniqueness of NE.

some sub-carrier is good for both users, and if player 1 gains a greater amount of resources, player 2 will gain a lesser amount. Thus, to maximize the network's total revenue, both players search for a fair distribution of the available resources (e.g., to obtain the minimal rate) via negotiation [35].

- 2) Players issue:  $K = \{1,2\}$  two CR players in MMC-CRN.
- 3) Agreement issue: Player 1 and player 2 receive their own utilities for their collaboration. Assume  $\pi^1$  and  $\pi^2$  are the utility functions for player 1 and player 2, respectively. In contrast, let  $\pi_{\min}^1$  and  $\pi_{\min}^2$  be the minimum utilities that both players are happy to receive; otherwise, they will not negotiate [37].
- 4) Modeling issue: 2-player BG is modeled by the pair of utilities for the two players and the minimum utilities that players received at the initial agreement. Let  $S$  be a compact and convex subset of  $\Re^K$  that represents the set of all possible utilities that players can obtain if cooperating via negotiation. Let  $\pi_{\min} = (\pi_{\min}^1, \pi_{\min}^2)$  be the initial agreement point. The pair  $(S, \pi_{\min})$  is called the 2-player bargaining problem [38], [39]. Moreover, the objective in this scenario is to maximize  $\pi^1$  and  $\pi^2$  concurrently.

Similar to NCGT, there is a common solution used for CGT, which is the topic of the following subsection.

### 3.3.2 Common Solution to CGT

One of the most commonly used solutions in CGT is called the Nash bargaining solution (NBS). This solution provides an optimal and fair resource allocation among players and can be defined as a function  $\bar{\pi} = f(S, \pi_{\min})$  that assigns a BG problem to a unique element of  $S$  based on the Nash axiom constraints<sup>7</sup> [36].

Moreover, the two important properties (i) existence and (ii) uniqueness are also associated with NBS and must also be examined in the problem.

**Theorem 3** “Existence and Uniqueness of NBS”: A unique and fair NBS  $f(S, \pi_{\min})$  can be obtained by maximizing a Nash product term based on

$$f(S, \pi_{\min}) = \arg \max_{\bar{\pi} \in S, \pi_i \geq \pi_{\min}^i \forall i} \prod_{i=1}^K (\pi_i - \pi_{\min}^i). \quad (3)$$

**Proof:** If the problem of resource allocation in MMC-CRNs is formulated as in (3), then the NBS satisfies all the Nash axioms and is shown to provide a fair and unique solution as presented in [37], [40].

<sup>7</sup> Please refer to [37] definition\_2 for addition details about the Nash bargaining intuitive axioms

### 3.3.3 Coalition Game

The second type of CGT is called a coalitional game and is abbreviated as *CG*. *CGs* have been shown to be an important tool for designing efficient, fair, and collaborative strategies in CRNs and can be divided into three categories<sup>8</sup> [41]: (i) canonical coalitional games, (ii) coalition formation games, and (iii) coalitional graph games.

*CG* theory describes how a set of players collaborate with each other by creating collaborating groups and can be defined as follows:

**Definition 5:** [36, 37], [41]: Let the set of players be denoted by  $K$  and a nonempty subset (coalition) by  $S$  (i.e.,  $S \subset K$ ). For a  $K$ -player game, any nonempty subset of players (i.e.,  $S$ ) is then called a coalition.

To demonstrate the idea of a  $K$ -player *CG*, we provide the following basic example:

**Motivating Example 2: “Modeling of a Coalition Game in MMC-CRNs”:** Consider the problem of subcarrier allocation in MMC-CRNs. The concept of a *CG* can be adopted to model  $K$ -players based on the following steps [37]: (i) Forming step:  $K$  players are grouped into pairs, named a coalition. (ii) Two-player negotiation step: Each coalition follows the procedures listed in (scenario 2) so that pairs in each coalition can negotiate with each other and exchange the information about available subcarriers. (iii) Reforming and convergence step: All the players are regrouped and continue their negotiation until convergence occurs.

## 4 Application of Supermarket Game in MMC-CRNs

In this section, we introduce the concepts of supermarket game theory, which includes the following: “pricing theory”, “auction theory”, and “oligopolistic competition” and their relevance to game theory.

### 4.1 Supermarket Game Theory: An Introduction

The concepts of a game as labeled in section (3) highlighted the following fact: *game theory provides mathematical tools to study the scenario where rational players interact with each other*. Based on this fact, game theory can be applied to a real supermarket scenario to study how individuals interact and negotiate with each other as buyers and sellers in the arena of a supermarket. The application of game theory to the market scenario is extremely interesting in the field of MMC-CNR for the following main reasons: (i) PUs enter the supermarket with the unused band as a commodity for sale to increase their revenue; and (ii) CRs, in contrast, enter the market looking for a commodity to buy (i.e., spectrum holes) to conduct a transmission with their partner. Thus, game theory can be applied to a spectrum supermarket to study the interaction among buyers and sellers accordingly.

<sup>8</sup> Please refer to [41] for a comprehensive survey on coalition game and the definition of its categories.

## 4.2 Pricing Theory

Pricing theory was first introduced and adopted in the arena of economics. In the field of spectrum market approaches, pricing theory becomes one of the important tools in the problem of resource allocation for the following reasons: (i) In the case of NCGT, pricing can provide an efficient NE by guiding selfish players to a more efficient operating point<sup>9</sup> [29]. (ii) In the case of CG, pricing can provide a better negotiation environment and fair distribution of the available resources so that the seller/buyers of the spectrum (i.e., PUs/CRs) are satisfied. (iii) Finally, pricing acts as a punishment technique for those buyers that generate certain amounts of interference to PUs, and, subsequently, interference to the owner of the spectrum can be minimized. To understand the general idea behind pricing, we have provided the following definition:

**Definition 6:** During spectrum trading in the distributed spectrum market, the pricing function can be defined as the cost that CRs must pay to PUs in terms of the performance degradation that may take place in the primary networks (PNs). However, in a certain scenario (e.g., centralized, downlink/uplink), CRs must buy spectrum for their transmission purposes, and, subsequently, prices are given to the PUBS as a cost of using the spectrum.

## 4.3 Auction Theory

Auction theory [42] is extensively used in the field of economics to determine, for example, the value of commodities that have uncertain prices. Recently, it has been applied to solve issues related to the problem of resource allocation in wireless networks. The common auction scenario can have the following components [43, 44]: (i) bidders, (2) a seller, (3) an auctioneer, and (iv) the commodity. Table 3 provides a mapping between basic components of auction theory and the entities of MC-CRNs.

**Table 4.** Mapping of auction theory elements to MC-CRNs.

Auction Component	Comments	Element of MC-CRN
Bidders	Bidders are the buyers (auction players) who want to buy resources for their transmission.	In MMC-CRNs, the buyers are CR nodes. Buyers compete with each other to obtain certain resource for transmission matters.
Seller	The seller is the owner of the radio resource (auction player) who wants to sell unused	In MMC-CRNs, the seller is a PUs/PUBS that holds the license to use the available spectrum.
Auctioneer	A midway agent between bidders and sellers who control the auction's process.	In MMC-CRNs, the auctioneer can be the seller (e.g., PUBS or PUs).
Commodity	The commodity or auction commodity is the product traded among buyers and sellers.	In MMC-CRNs, the commodities are, for example, the vacant subcarriers.

<sup>9</sup> Please refer to [29] for an example on how pricing method plays an important role in providing efficient NE.



Generally speaking, the auction supermarket adopts the following scenario: (i) The buyers compete with each other by submitting an (*ask*) asking about the price of the product to be sold in the spectrum market to obtain one of the available commodities. (ii) The sellers, in certain scenarios, compete with each other to obtain additional buyers to increase their revenue by submitting a (*bid*) indicating the bidding price for the requested product [43]. Hence, game theory is the best mathematical tool to analyze the behavior of sellers, buyers and auctioneer in an auction scenario. Accordingly, the application of an auction as a game has generally been adopted in the problem of resource allocation in CRNs. In the following paragraph, we provide a scenario of the application of an auction game in MMC-CRNs as shown below.

**Scenario 4 “Application of Auction Game in MMC-CRNs”:** An auction game can be adopted in MMC-CRNs to encourage both sellers and buyers in the network to select their strategies wisely to achieve the network’s objectives [43]. The main functions of auction game scenario are as follows:

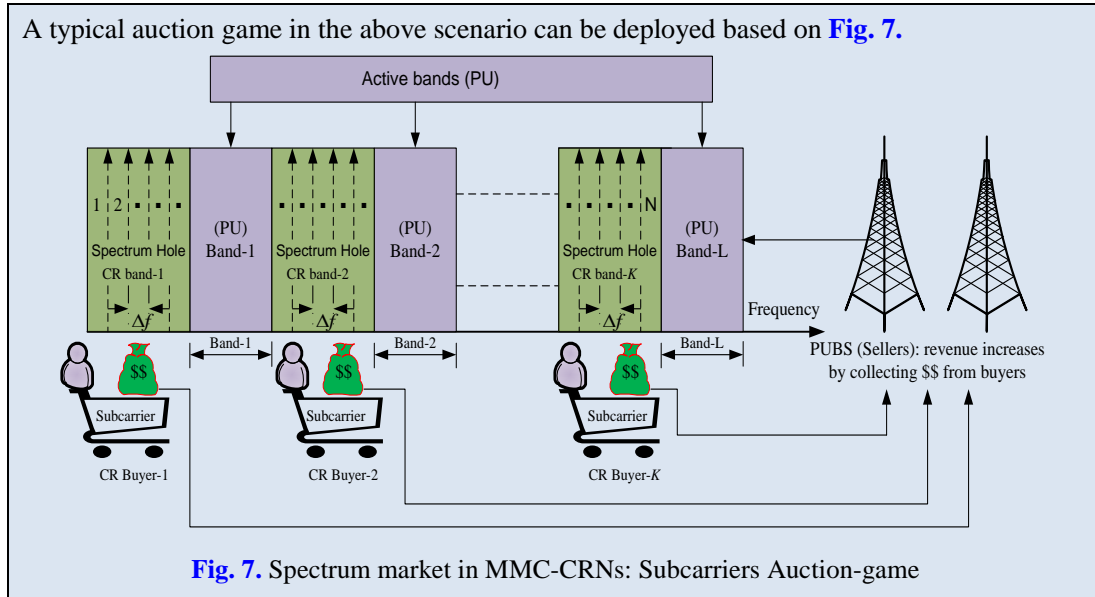
- 1) Players  $K = \{1, 2, \dots, K\}$ : Players in the auction game are both sellers (PUBs) and buyers (CRs).
- 2) Players’ strategies: The action set of each player is a set of *asks* submitted by rational buyers asking for available subcarriers and a set of *bids* submitted by rational sellers informing buyers about the cost for the available subcarriers.
- 3) Game objective (payoff): The objective of the auction game is to reach an equilibrium strategy among buyers and sellers, where buyers are satisfied with the cost and the sellers are satisfied with the profit.

Auction solution: The NE and its details (i.e., existence and uniqueness) are considered to be a common solution in auction games.

...Continue to next page for *Scenario*

3

In Fig. 7, the buyers are the CR nodes, the seller is the primary user base stations (PUBS) and the auctioneer is the PUBS itself. The sellers offer the unused subcarrier to the buyers at a certain price to increase their revenue. The buyers can accept the offer and make their transmissions accordingly.



#### 4.4 Oligopoly Market Competition

When a small number of firms compete with each other to maximize their revenue by managing the quantity or the price of the offered commodity, then the market can be called an “oligopolistic market” [45] with the following assumptions [46]: (i) few firms are available in the market; (ii) the firms compete with each other independently to increase their revenue; and (iii) each firm should take into account the available strategies of other firms in the market.

Moreover, the behavior of firms in an oligopoly market (i.e., interaction and competition) can be modeled using the concept of game theory. However, modeling an oligopoly market as a game requires different models that have different supermarket structure and different strategies [47]. Table 5 summarizes features of the most familiar oligopoly game in the literature. To facilitate a better understanding to the concept of oligopoly market game, we have provided the following example:

##### **Motivating Example\_3 [50]: “Modeling of Oligopoly-Bertrand Game in MMC-CRNs”**

Assuming that  $L$ -PU spectrum service providers compete with each other in an MMC-CRN oligopoly-Bertrand scenario, the resource allocation problem can be described based on the following: (i) commodities are the vacant subcarriers offered by PU spectrum providers, (ii) firms (i.e., players) are the spectrum providers that compete with each other to obtain additional buyers (i.e., CRs) to maximize their profit, (iii) consumers are the CRs that willing buy/rent good commodities (i.e., subcarriers with less interference to PU) at a reasonable price, (iv) strategies of the firms are related to the supplied quantity or offering price, (v) the

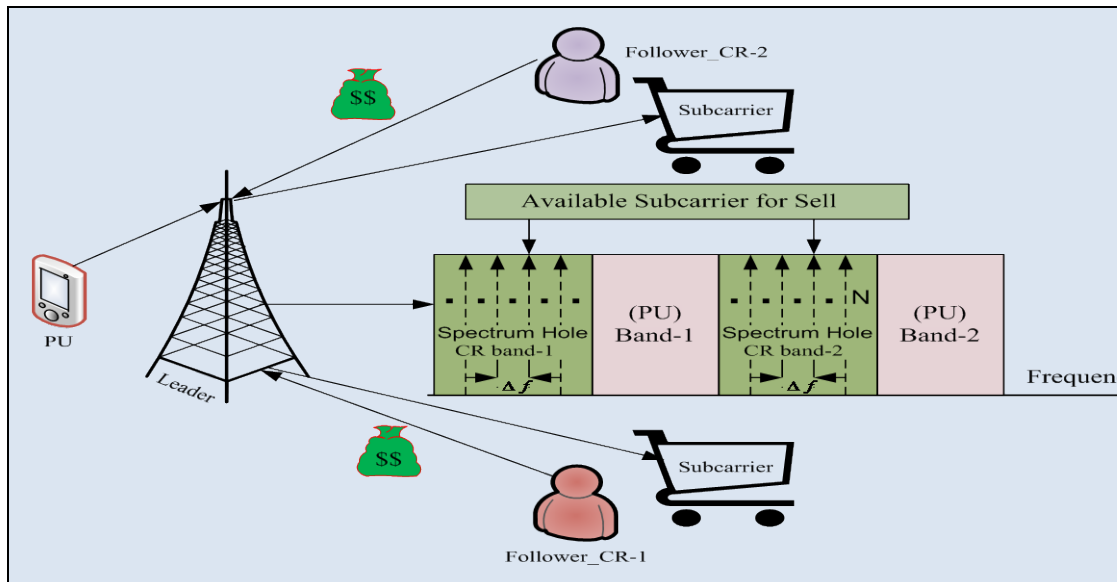
payoff of a firm is linked by its surplus function (revenue minus cost) for renting vacant subcarriers to CRs and (vi) the game solution is the NE.

**Table 5.** Different types of oligopoly games.

Game	Key Features	Example
Cournot	<ul style="list-style-type: none"> <li>Firms make their decisions simultaneously.</li> <li>Single commodity produced by <math>n</math> firms.</li> <li>The actions of firms affect other firms in the market because the price depends on the overall quantity.</li> </ul>	[48], [49]
Bertrand	<ul style="list-style-type: none"> <li>Firms' actions are performed independently and simultaneously in terms of price.</li> <li>Firms' decisions are strategic variable prices instead of quantities output.</li> </ul>	[50]
Stackelberg	<ul style="list-style-type: none"> <li>Firms choose their quantities sequentially.</li> <li>Leader-follower approaches: Because the decisions of firms are sequential, the firm that produces his offer first is called the leader, and the others are followers that take into account the leader's decisions to make their own decisions.</li> </ul>	[51]

**Motivating Example\_4: “Modelling of stacklberg in uplink MMC-CRNs”**

Assuming an uplink scenario in OFDMA based CRNs as shown in Fig.8



**Fig. 8.** Stacklberg Game in MMC-CRNs

The PUBS is the owner of the spectrum and their users (i.e., PUs) transmit to the PUBS for free of charge. CRs, in contrast, need to pay to the PUBS in order to get subcarrier for their needs. The strategy between the PUBS and CRS can be modelled according to Stacklberg market game with the following assumptions: (i) PUBS is the leader of market game, (ii) PUBS sell its vacant band and charge a price for each CR to maximize its profit, (iii) the CRs are the followers in this scenario and need to follow the pricing policies generated by the PUBS, and (iv) after all prices distributed by PUBS, the CRs make a decision to utilize the subcarrier with controlled power to maximize their utility function based on NCGT.

Note that the same example can be simply applied to Cournot and Bertrand model by following the features listed in [Table 5](#).

## 5 Resource Allocation in MMC-CRNs

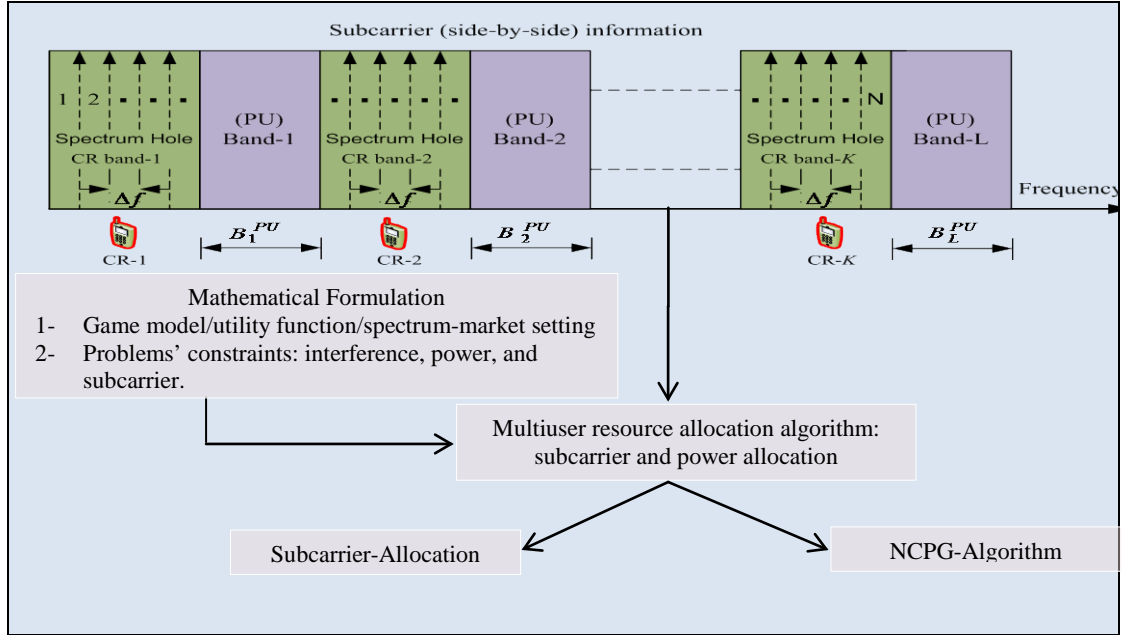
### 5.1 Resource Allocation in MMC-CRNs: An Overview

The resource allocation problem (e.g., power and subcarrier allocation) in MMC-CRNs brings to academics certain challenges because of the following facts: (i) two different mobile radio users (i.e., PUs and CRs) interact with each other, transmitting independently within the same band, and may be based on different standards [16], (ii) CRs are rational, aiming to allocate their resource independently and, in some scenarios, selfishly and (iii) the interference that arises from CRs-to-PUs and vice versa is another concern that must be treated carefully in MMC-CRNs.

The first two points make game theory a promising tool for resource allocation in CRNs because game theory is extensively applied to study situations with conflicting interests. Moreover, in multicarrier techniques (e.g., OFDM and FBMC), the PUs leave some unused subcarrier during their idle period. Thus, CRs have the opportunity to utilize unused subcarriers. In another words, the CR pays the owner of the spectrum to temporarily rent his vacant bands. Therefore, a mutual benefit exists in this scenario, where the rental users (i.e., CRs) take the advantages by utilizing the vacant spectrum for their transmission purposes, and the owner of the spectrum can maximize their revenue accordingly [3]. This basic concept makes adopting market theory in the problem of resource allocation another promising solution. In addition, an overview of the problem of resource allocation in MMC-CRNs is shown in [Fig. 8](#) and can be described based on the following example.

**Motivating Example\_5: “Resource allocation problem in OFDMA-CRNs”:** Assuming that  $K$ -CR players are available in the network with  $N$  OFDM subcarrier frequency distributions as shown in [Fig. 8](#). Thus, the problem is to structure the following: (i) Problem formulation, which includes the design of the utility function (e.g., rate maximization) with a set of constraints (e.g., channel/power/interference constraints). (ii) Subcarrier allocations, abbreviated as **SA**, (i.e., subcarriers to CRs allocation matrix  $\mathbf{SA} = [sa_{k,n}]_{K \times N}$ ) where each subcarrier is assigned to only one CR. (iii) Non-cooperative power allocation game,

abbreviated as **NCPA**, for each subcarrier (i.e.,  $\mathbf{PA} = [\mathbf{p}_n]_{N \times 1}$ ), which can be determined based on maximum power and interference constraints.



**Fig. 8.** Overview of resource allocation problem in MMC-CRNs

The optimal solution to the resource allocation problem in MMC-CRNs is, in general, an NP hard problem. Thus, the suboptimal scheme is preferred in such a scenario and can be achieved by decomposing the resource allocation problem into two sub-problems (i.e., an **SA** problem and a **NCPA** problem). Moreover, the optimal **NCPA** algorithm can be achieved via a Lagrangian technique.

Furthermore, the problem of resource allocation in MMC-CRs can take the following three forms: (i) uplink resource allocation with multiple local power constraints (e.g., see [12], [52]-[53]); (ii) downlink resource allocation with global power constraints (e.g., see [54]-[55]); and (iii) distributed resource allocation as in an ad-hoc scenario (e.g., see [56]-[57]).

## 5.2 Resource allocation in MMC-CRNs via game/supermarket game theory

Studies on resource-allocation-based game theory in multicarrier wireless networks can be divided into two general approaches [58]: (i) rate-adaptive games as in [59] and (ii) margin-adaptive (i.e., MA) games as in [60]. However, another approach can be used when using game theory and supermarket game theory: (iii) spectrum-market game, which can be either rate-adaptive, margin-adaptive or a pure spectrum market as in [45], where CRs are required

to follow certain rules to obtain acceptable commodities (i.e., frequency spectrum), while the owners of the market increase their revenue accordingly.

In the case of rate-based resource allocation, the problem is normally formulated to maximize the total rate of the network subject to subcarrier, power and interference constraints as shown in Fig. 9, whereas in margin-based resource allocation, the problem is formulated to minimize the total power subject to subcarrier and quality of service requirement for each user in the network.

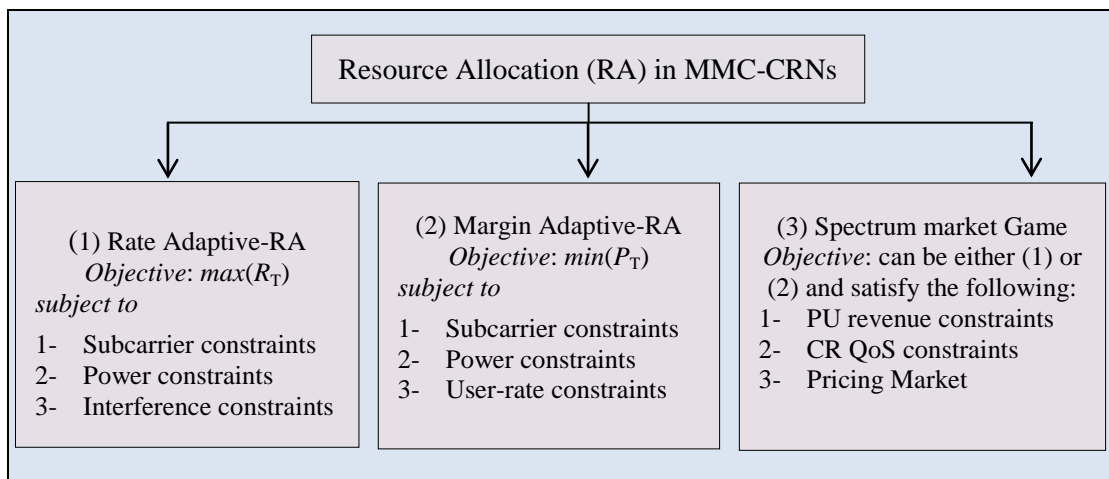


Fig. 9. Taxonomy of resource allocation problems.

For the scenario of MMC-CRNs, most of the studies conducted in the literature focused on rate-adaptive compared to margin-adaptive classes because the latter technique makes the optimization problem more complex compared to the rate adaptation technique. Furthermore, the spectrum market game in the problem of resource allocation in MC-CRNs has received light attention, and more effort is needed in the field of spectrum market game theory.

### 5.3 Discussions on related works

In this tutorial, we have classified the studies conducted in the literature into five classes as shown in Table 6.

**Note 1:** According to Table 6 class (B), the problem of resource allocation in multicarrier *non-CR* scenarios is presented (i) to allow the readers to become familiar with the concept of resource allocation in multicarrier methods using both game and supermarket game theory and (ii) to show that moving from the non-cognitive scenario to the cognitive model can be easily performed by taking into account the interferences generated from CRs-to-PUs (i.e., eq. 1) and the interference generated from PUs-to-CRs (i.e., eq. 4 in Appendix I) accordingly.

**Table 6.** Details of applications' classes

Class Type	Class description	Sub-class description
A	Research type	A.1: Technical research A.2: Survey research
B	Cognitive and non-cognitive scenario	B.1: Cognitive scenario B.2: Non-cognitive scenario
C	Resource allocation approaches	C.1: Rate-adaptive game C.2: Margin-adaptive game C.3: General Approach
D	Cell model	D.1: Single-cell scenario D.2: Multiple-cell scenario
E	Network model	E.1: Centralized model E.2: Distributed model

In addition, to add to the presentation for the mentioned classes, we have provided *scenario 5* to further assist readers. Moreover, all the related studies in the literature were classified similar to the details listed in *Scenario 5*.

**Scenario 5:** Applications' classes: We assume that we have been asked to analyze the problem of resource allocation in multicarrier wireless network based on the following assumptions:

Research Type	Problem	Problem Approach	Cell Model	Network Model
Tutorial	Resource allocation in MMC-CRNs	General approach (Tutorial)	Single cell	Distributed

Based on **Table 5**, the above mentioned assumptions can be translated to the following codes:

Research Type	Problem to solve	Problem Approach	Cell Model	Network Model
A.2	B.1	C.3	D.1	E.2

Following the listed classes in **Table 6**, we have provided a summary of studies related to the resource allocation problem in multicarrier technology based on game and supermarket game model as shown in **Table 7**.

In the following sub-sections, we demonstrate the main features of related studies conducted in the literature.

**Table 7.** A summary of related studies on the applications of game and supermarket game theory in multicarrier techniques.

Ref.	Game Model	Class											Game Solution
		A		B		C			D		E		
		A.1	A.2	B.1	B.2	C.1	C.2	C.3	D.1	D.2	E.1	E.2	
[43]	Auction-Game		✓	✓				✓	✓		✓	✓	➤ Game-Solutions: different solutions can be provided in auction game according to auction design such as: NE, mixed-strategy equilibrium, correlated equilibrium (CE)
[59]	NCGT	✓			✓		✓		✓			✓	➤ Game-Solution: NE
[60]	Market-Game	✓		✓				✓	✓		✓		➤ Game-Solution: NE
[62]	Auction-Game	✓			✓	✓			✓	✓	✓	✓	➤ Game-Solution: NE
[63]	CGT	✓			✓	✓			✓		✓		➤ Game-Solution: NBS
[64]	NCGT	✓		✓		✓				✓	✓		➤ Game-Solution: NE
[65]	NCGT	✓			✓	✓			✓		✓		➤ Game-Solution: CE
[66]	NCGT	✓			✓	✓				✓	✓		➤ Game-Solution: NE in Potential game
[67]	Potential-NCGT	✓		✓		✓			✓			✓	➤ Game-Solution: NE
[68]	Market-Game	✓		✓		✓			✓		✓		➤ Game-Solution: different solutions depending on market setup.
[69]	Market-Game		✓	✓				✓	✓		✓	✓	➤ Game-Solution: different solutions depending on market setup.
[70]	Stochastic-Market-Game	✓			✓	✓			✓		✓		➤ Game-Solution: NE
[71]	Auction-Game	✓			✓	✓				✓	✓		➤ Game-Solution: NE
[72]	NCG-Pricing	✓		✓		✓			✓			✓	➤ Game-Solution: NE
[73]	NCG-Colonel Blotto-Game	✓		✓		✓			✓		✓		➤ Game-Solution: NE
[74]	Market-Game	✓		✓				✓		✓		✓	➤ Game-Solution: NE with pricing
[75]	Stackelberg-Market Game	✓		✓		✓			✓			✓	➤ Game-Solution: NE with pricing
[76]	NCGT	✓		✓		✓			✓			✓	➤ Game-Solution: NE with pricing
[77]	Oligopoly Market	✓		✓				✓	✓		✓		➤ Game-Solution: NE with pricing



### 5.3.1 Game and supermarket game in non-cognitive MC scenarios

Game and supermarket game theory have been recently adopted to address the problem of resource allocation in wireless networks (see, for example, [59], [63], [64], [66], [70] and [71]). **Table 8** summarizes the main features of a number of selected studies conducted on the resource allocation problem in multicarrier wireless networks using both game theory and supermarket game theory.

**Table 8.** Main features for RA in MC-wireless scenarios.

[ ]	Objective	Players	Strategies	Payoffs and Constraints
[59]	Reduce power consumption of the network and satisfy the target rate on each subcarrier in OFDM networks via <i>NCGT</i> .	Nodes	Players to allocate power and subcarrier.	Cost function: Minimizing transmitted power subject to local power and rate constraints.
[63]	Efficient and fair RA in multiuser OFDM networks through <i>CGT</i> .	Nodes	Players to allocate power and subcarrier.	<i>BG</i> utility function: Maximize bargaining utility function subject to power, rate and channel constraints.
[65]	Energy efficient (EE) RA in OFDMA to balance the trade-off between total energy efficiency and fairness via <i>NCGT</i> .	Subcarriers	Subcarriers to choose most satisfying users.	Logarithmic utility function: Maximizing EE utility function subject to power constraints.
[66]	Fast, distributed RA in OFDMA networks via potential <i>NCG</i> .	Nodes	Players to select power and subcarrier.	Cost function: Maximizing surplus function subject to power constraints.
[70]	Fast, distributed, joint RA algorithm in uplink OFDMA networks via potential <i>NCG</i> .	Nodes	Players to select power and subcarrier.	Cost function: Maximizing surplus function subject to maximum power constraints.
[71]	Efficient RA in combinatorial auction game in multi-cell OFDMA networks.	Nodes and base stations (BS)	Nodes' strategies: subcarrier and power allocation. BSs' strategies: setting the price for the commodity.	Cost function: Minimizing transmitted power subject to local power, rate and channel constraints.

### 5.3.2 Cognitive Radio MMC Scenario

In the case of MMC-CRNs, game theory and supermarket game were proven to provide efficient and effective spectrum sharing among CRs and the owner of the spectrum because they can properly define the interaction and competition among players [47]. There are a number of studies that adopted game and supermarket game theory in multicarrier CRNs to address the related issues to spectrum access (i.e., power, subcarriers and rate) as in [43], [61], [64], [67-69],[72-76]. To be more specific we provide a review to a number of related studies conducted in the literature as shown below.

In [61], a spectrum monopoly-market scenario based on non-cooperative game theory was proposed for OFDM-based CRNs. The most interesting point in this work is that the CRs performing two interesting tasks as follows: (i) using part of the leased band to help the PUs by relaying tier data from the source to destination and (ii) use the remaining part of the leased band for their own activities. Moreover, the PUs enters the market aiming to sell certain amount of its vacant band to CRs and the CRs, in contrast, enters the market aiming to transmit with optimal power in order to fulfil the above mentioned tasks. The authors adopted non-cooperative game in order to find optimal power for the CRs in the leasing-market scenario.

Instead of Stackelberg game, the authors employed auction to solve the spectrum leasing scenario where the CRs are involved in the leasing decisions which consider another interesting contribution of this work among others in the literature. The authors guided the readers to some selected references for the mathematical verification of the existence and uniqueness of the NE. However, the convergence of the proposed algorithm to a stable point is a bit slow. Hence it is not appropriate for more particle scenario.

More complicated RA scenario in MMC-CRNs based on non-cooperative game has been proposed in [64]. Instead of multiuser-single cell scenario, the authors proposed RA algorithm in multiuser multi-cell MC-CRNs which resulted in NP-hard problem. To tackle this problem, the authors adopted the multiple access channels (MAC) technique in order to convert their problem to a concave optimization problem which is one of the novel contributions of this work.

Non-cooperative game theory based on MAC technique has been adopted in this work to allocate the subcarrier and power in uplink scenario. Existence and uniqueness of the NE are validated mathematically. However, the authors didn't provide any evidence to show that the proposed power algorithm convergence to unique NE via simulation. Moreover, the cheating scenario among the selfish CRs has been ignored in this work.

One of the most important problems to tackle while allocating subcarriers among CRs in MC technique is how to mitigate the generated interference to the owner of the spectrum (i.e., PUs). This significant problem has been considered in [67] in order to designed not only efficient RA algorithm but also optimal subcarrier allocation with minimal interference in MC-CRNs. Firstly, the authors adopted an interference mitigation objective in the utility function and defined the potential function where the NE is always guaranteed. Secondly, the authors proposed modified subcarrier-game scheme know as autonomous number of subcarrier selection (ANSS) method which considered as etiquette provider for the whole network. Through ANSS scheme the players are allowed to have some interaction among them before the start of the real game. This makes each player aware of its environment and the available recourses and can easily measure the

amount of interference from neighbor players within its radius of interference. The novelty of this work comes from introducing potential game with means of cooperation and self-awareness in the player's utility function and introducing sequential best response play in order to make the ANSS-game model converge to unique stable point (i.e., NE). However, the proposed algorithm shows slow convergence to a stable point which is the main drawback for the proposed algorithm.

An overlay spectrum sharing based on game-pricing approach in MC-CRNs has been considered in [72]. The main contribution of this work is by adopting pay-off function that comes with two parts: (i) rate-based utility function and (ii) pricing function. The pricing function, in contrast, composed of two parts in order to manage: (i) the interference generated among CRs in the network represented by normal pricing function, and (ii) the negative effect from active CRs to PU's sub-band represented by exponential pricing function. Accordingly, sufficient and fair spectrum sharing can be achieved in this work by provide adequate protection to the PUs. Furthermore, the existence and uniqueness of the proposed objective function has been verified mathematically and via simulation as well. The authors considered a distributed scenario; however the cheating scenario among the selfish CRs has been ignored in this work. Unlike [61] and [67] the proposed algorithm in [72] resulted in a fast convergence to the NE. Hence, it is more appropriate for more practical scenario.

Resource allocation in MC-CRNs based on market-game has been considered in [73]. The most interesting issue in this work, among others, is that the authors adopted Colonel Blotto market game to model the problem of subcarrier and power allocation in both uplink and downlink scenario. Unlike [64], the authors proposed a simple optimization problem by adopting interference temperature constraint instead of global power constraint and Blotto game used to allocate the resources among the players which resulted in a fair allocation and fast convergence to NE. Moreover, the cheating scenario has been introduced in this work which is another obvious issue in this work compared to that in [72]. The existence and uniqueness of the NE have been verified mathematically and via simulation as well. However, the convergence of the proposed algorithm is slightly slower than that in [72].

In [74], the authors proposed new and dynamic pricing scheme in a competitive spectrum-supermarket. The noticeable advantages of this work compared to other studies related to spectrum market, is that the buyers play an important role in the convergence of the market by evaluating the spectrum sellers in a different way based on the quality of goods provided by sellers and the prices offered by the sellers. The sellers, based on buyer's evaluation, are trying their best to show the available spectrum at affordable prices to attract not only quality sensitive buyers but also the price wise buyers. Therefore, the performance of the spectrum market can be improved accordingly. Game theory has been adopted in [74] to analyze the profit of the sellers. Convergence of the market has been well investigated by simulation and the market convergence to stable points where all buyers and sellers are satisfied in their commodities and profit respectively. However, the proposed market mode is not practical in heterogeneous spectrum market where many buyers and sellers are available because of the long time required by the buyers to evaluate different goods with different prices provided by different sellers.

The authors in [76] proposed an energy-efficient algorithm for joint power and subcarrier allocation in the uplink MC- underlay CRNs based on pricing-game model. The objective of the

proposed game model is to maximize the EE utility function and to guarantee the PU' QoS. To fulfil this objective, the authors adopted a linear bandwidth-pricing scheme to improve the efficiency of the NE. Unlike [72], where the authors proposed non-linear pricing scheme, authors in [76] proposed a linear pricing function which reduced the complexity of the optimization problem. Both uniqueness and existence of the NE have been proved mathematically. However, the authors didn't show the convergence of their algorithm in the simulation results. The selfishness scenario has been considered in this work. However, the cheating scenario which considered an important scenario to consider especially in EE paradigm has been ignored. Hence it is not sure how apply the proposed distributed algorithm in more realistic EE scenario.

In addition to the above review, Table 9 summarized the main features of the related studies conducted in the literature to solve the problem of RA in MMC-CRNs with the aid of game and supermarket game theory. This will help the interested readers to memorize the most important components of game/market theory which are: *players*, *strategies* and *utility function* and how these components are related to the objective of the game/market scenario.

**Table 9** Main features for RA in MC-wireless scenarios.

[ ]	Objective	Players	Strategies	Payoffs and Constraints
[61]	Efficient, cooperative communication in OFDM spectrum monopoly leasing market	CRs: bidders PUs: auctioneer	CRs: providing relaying service to PUs. PUs: leasing vacant band as a revenue of cooperation to the CRs	CRs: Maximize profit function as a function of revenue that can be obtained and the power consumed in the cooperation process. PUs: Maximize its data rate by getting more relay services from CRs.
[64]	RA algorithm in multi-cell multiuser uplink scenarios in FBMC CRNs	CRBS	CRBS: choosing power on different bands for their own users (CRs)	Maximizing information rate for each CRBS subject to interference and power constraints.
[67]	Interference minimization algorithm in OFDMA CRNs	Nodes	Subcarriers selections	Potential game utility function subject to interference, power and subcarrier constraints
[68]	Spectrum leasing market for better spectrum utilization in OFDMA CRNs	CRs: bidders PUs: auctioneer	CRs: providing relaying service to PUs. PUs: selecting best relaying services	Maximizing rate function subject to power, interference and QoS constraints.
[72]	Distributed, non-cooperative power allocation in FDMA CRNs	Nodes	Allocating power on subcarriers with less interference to PUs	Cost function: maximizing surplus function subject to maximum power, interference and channel constraints.
[73]	Joint subcarrier and power allocation in OFDMA CRNs via Colonel Blotto	Bidders (CRs)	Power and budget allocation	Utility function subject to power, budget and interference temperature constraints.

[74]	Competitive spectrum supermarket with dynamic prices	Service providers (sellers) and nodes (buyers)	Sellers: attract potential consumers. Consumers: evaluating (and selecting) sellers with good applications, operating technologies and location	Dynamic pricing for price- and quality-sensitive customers. Moreover, sellers group against others to increase their revenue.
[76]	Energy-Efficient (EE) resource allocation in OFDMA CRNs with pricing game	CRs	Joint subcarrier and power allocation	Utility function: maximize the EE utility function subject to bandwidth and power constraints

Finally, it is worth mentioning that in [43] and [69], the authors provide a comprehensive survey for the application of auction and spectrum leasing in CRNs, respectively, and discuss fundamentals and concepts of supermarket game theory.

## 6. Open Research Directions

Game and supermarket game theory were proven to be an effective tools in analyzing the problem of resource allocation in MMC-CRNs. However, there are still a number of shortcomings in certain areas where game and supermarket game must attract more attention as follows:

- 6.1 MC-CR relay game networks<sup>10</sup>: Cooperative communication with the aid of relay nodes in MC-CRNs has been recently acknowledged as a promising technology in dynamic spectrum sharing because of the assured enhancement in the performance of both CRs and PUs by providing flexible and efficient resource allocation among all the residents (i.e., players) in a given network. Research on MC-CR relay networks has recently been conducted in the literature, see for example, [79]-[81]. However, extensive studies have not been conducted on the problem of resource allocation in MC-CR relay networks using the concept of game and supermarket game theory. Thus, the problem of resource allocation in MC-CR relay networks remains an open issue that must be addressed.
- 6.2 Multi-cell MC-CRNs: Most of the studies in the literature focused on single-cell scenarios to address the problem of subcarrier and power allocations among players caused by the simplicity associated with single cell scenario. However, to provide a more practical scenario for real applications, multi-cell scenarios should be considered. Game theory and supermarket game theory can be adopted to facilitate the scenario of multi-cell MC-CRNs in the following proposed scenario: (i) Network-user game/market: In this scenario, the network has the opportunity to select good players that follow the network renting policies (i.e., renting spectrum with minimal interference to PUs) while punishing the players with high spectrum renting prices. (ii) User-network game/market: In this scenario, the users have the opportunity to choose the network that offers a good commodity with a good QoS.

<sup>10</sup> Please refer to [78] for examples of the application of game theory to interference coordination in OFDMA relay networks.

- 6.3 CRNs over 4G networks: Cognitive radio has become one of the most recognized technologies in 4G networks, e.g., Long Term Evolution Advanced (LTE-A), to solve the spectrum insufficiency problem [8]. OFDM, in contrast, is chosen as the PHY air interface for 4G downlink transmissions because of its promising advantages in providing proper performance and flexibility in allocating spectrum. Thus, adopting game theory and supermarket game theory in the problem of resource allocation in MC-4G-based cognitive radios can be an interesting direction of future study.
- 6.4 Green Spectrum-Market Game: Research on green technology wireless networks has received significant amounts of attention recently and has become a very important area of investigation for the improvement of the energy efficiency (EE) of networks. In fact, a number of researchers investigated the problem of resource allocation in EE-MMC-CRNs (see, for example, [82]). However, the application of game theory and supermarket game theory to the problem of resource management in MC-Green-CRNs is ignored in recent studies, and a significant amount of effort is needed in this significant field of research. Game theory and supermarket game theory play important roles in designing green CRNs. By applying game theory and supermarket game theory, CR nodes can avoid transmission on certain spectrum that is frequently used by PUs. Furthermore, a green spectrum market can be achieved by applying intelligent supermarket game theory, where CRs incorporate a wide variety of activities, including renting the vacant spectrum wisely by informing other CRs about renting policies (e.g., pricing) so that CRs have the ability to rent spectrum holes that provide minimal interference to nearby PUs. Moreover, by applying a green spectrum market, the CRs can change their expense behavior by buying from good sellers that offer reasonable price for the available spectrum with a good QoS.

## 7. Conclusion

Game theory and supermarket game theory have become promising tools for modeling and analyzing the interactions of CRs in the context of resource allocation problems in CRNs. In this article, we have presented a comprehensive tutorial on the concepts and applications of “game theory” and “supermarket game theory” to the problem of resource allocation in MMC-CRNs. The game model, in this article, is categorized based on the behavior and interactions among CRs and PUs as non-cooperative, cooperative and supermarket games. In addition, a set of definitions, examples and scenarios related to each model were presented in this tutorial to facilitate an understanding of the concepts of game and supermarket game theory accordingly. The similarities between the behavior of players in MMC-CRNs and the interactions among people in a real market make supermarket game theory better suited to analyze the spectrum trading in MMC-CRNs. However, research on the applications of game and supermarket game theory in MMU-CRNs remains in its infancy, and more problems must be investigated properly. Finally, we hope that this tutorial will provide important information for interested researchers in the areas of game and supermarket game theory.



## 8. Acknowledgements

The authors are very appreciative to the Universiti Kebangsaan Malaysia and Malaysia's Ministry of Higher Education ([ERGS/1/2013/ICT03/UKM/02/1](#)) for funding of this work. The authors would also like to thank the editor and the anonymous reviewers for their valuable feedback.

## Appendix

### Appendix I: MMC-CRNs interference' mathematical formulation

In equation (1) we have showed the generated interference from CRs-to-PUs in side-by-side spectrum allocation. In addition to eq.1, there is another amount of interference that generated from PUs-to-CRs which can be defined and modelled according to

**Definition 10 [19-21]:** interference generated by  $l$ -th PU to the  $n$ -th OFDM subcarrier occupied by  $k$ -th CR denoted by  $I_{l,k}^{CR}$  can be defined as the integration of the PSD of the  $l$ -th PU across the  $n$ -th subcarrier.

Mathematically speaking the interference introduced by PU's signal can be modeled according to

$$I_{l,k}^{CR} = \int_{d_i + \Delta f/2}^{d_i + \Delta f/2} g_k^p \phi_k^{PU}(f) df \quad (4)$$

where  $\phi_k^{PU}(f)$  is the PSD of the  $l$ -th PU and  $g_k^{CP}$  is the channel gain of the interference link from the PUBS to the  $k$ -th CR on the  $n$ -th subcarrier.

## References

- [1] FCC, ET Docket No 03-237 Notice of inquiry and notice of proposed Rulemaking, November 2003. ET Docket No. 03-237.
- [2] S.Haykin, "Cognitive Radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005. [Article \(CrossRef Link\)](#).
- [3] T. A. Weiss and F. K. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *IEEE Commun. Magazine*, vol. 42, pp. S8-14, 2004. [Article \(CrossRef Link\)](#)
- [4] Elias Z. Tragos, Sherali Zeadally, Alexandros G. Fragkiadakis, and Vasilios A. Siris, "Spectrum Assignment in Cognitive Radio Networks: A Comprehensive Survey," *IEEE Commun. Surv. Tut.*, vol. 15, no. 3, Third Quarter 2013. [Article \(CrossRef Link\)](#)
- [5] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, vol. 50, pp. 2127-2159, 2006. [Article \(CrossRef Link\)](#)
- [6] I. F. Akyildiz, W.-Y. Lee, and K. R. Chowdhury, "CRAHNs: Cognitive radio ad hoc networks," *Ad Hoc Networks*, vol. 7, pp. 810-836, 2009. [Article \(CrossRef Link\)](#)
- [7] Z. Qing and B. M. Sadler, "A Survey of Dynamic Spectrum Access," *Signal Processing Magazine, IEEE*, vol. 24, pp. 79-89, 2007. [Article \(CrossRef Link\)](#)

- [8] Songlin Sun, Yanhong Ju, and Yasushi Yamao, "Overlay Cognitive Radio OFDM System For 4G Cellular Networks," *IEEE Wireless Communi.*, Apr.2013. [Article\(CrossRef Link\)](#)
- [9] T. C. Clancy, "Formalizing the interference temperature model," *Wireless Communications and Mobile Computing*, vol. 7, pp. 1077-1086, 2007. [Article\(CrossRef Link\)](#)
- [10] N. Devroye, M. Vu, and V. Tarokh, "Cognitive radio networks," *IEEE Signal Process. Magazine*, vol. 25, pp. 12-23, 2008. [Article\(CrossRef Link\)](#)
- [11] B. Farhang-Boroujeny and R. Kempter, "Multicarrier communication techniques for spectrum sensing and communication in cognitive radios," *IEEE Commun. Magazine*, vol. 46, pp. 80-85, 2008. [Article\(CrossRef Link\)](#)
- [12] Shaowei Wang, Mengyao Ge, and Wentao Zhao, "Energy-Efficient Resource Allocation for OFDM-Based Cognitive Radio Networks," *IEEE Transc. Communi.*, vol.61, no.8, Aug.2013. [Article\(CrossRef Link\)](#)
- [13] Ibrahim Budiarto, Homayoun Nikookar, and Leo P. Lighart, "Cognitive Radio Modulation Techniques," *IEEE signal processing magazine*, Nov.2008. [Article\(CrossRef Link\)](#)
- [14] Musbah M R Shaat, "Resource Management in Multicarrier Based Cognitive Radio Systems," *PhD Thesis*, Universitat Politècnica de Catalunya (UPC), Jan. 2012.
- [15] P.Amini, Roland Kempter, Rong Rong Chen, Lekun Lin and Behrouz Farhang-Boroujeny, "Filter Bank Multitone: A Physical Layer Candidate for Cognitive Radio," in *Proc. of Software Defined Radi Tech. Conf.*, Nov. 14-18, 2005.
- [16] B. Farhang-Boroujeny, "OFDM Versus Filter Bank Multicarrier," *IEEE Signal Process. Magazine*, vol. 28, pp. 92-112, 2011. [Article\(CrossRef Link\)](#)
- [17] Sriram N. Premnath, Daryl Wasden, Sneha K. Kasera, Neal Patwari, and Behrouz Farhang-Boroujeny "Beyond OFDM: Best-Effort Dynamic Spectrum Access Using Filterbank Multicarrier," *IEEE/ACM TRANSACTIONS ON NETWORKING*, VOL. 21, NO. 3, JUNE 2013. [Article\(CrossRef Link\)](#)
- [18] Timo Weiss, Joerg Hillenbrand, Albert Krohn, Friedrich K. Jondral, "Mutual Interference in OFDM-based Spectrum Pooling Systems," in *Proc. of IEEE Vehicular Tech. Conference, VTC* May 2004. [Article\(CrossRef Link\)](#)
- [19] G. Bansal, Md. Hossain, and V. K. Bhargave, "Optimal and Suboptimal Power Allocation Schemes for OFDM-Based Cognitive Radio Systems," *IEEE Transc. Wireless Communi.*, Vol.7, no.11, Nov.2008. [Article\(CrossRef Link\)](#)
- [20] Shaowei Wang, Nanjing, Fangjiang Huang, and Zhi-Hua Zhou, "Fast Power Allocation Algorithm for Cognitive Radio Networks," *IEEE Communi. Letter*, vol.15, no.8, Aug. 2011. [Article\(CrossRef Link\)](#)
- [21] M. Shaat, and F. Bader, "Efficient resource allocation algorithm for uplink in multicarrier-based cognitive radio networks with fairness consideration," *IET Communi.*, vol.5, no.16. 2011. [Article\(CrossRef Link\)](#)
- [22] O. M. J.V.Neumann, "Theory of Games and Economic Behavior," *Princeton University Press*, 1944.
- [23] G. Scutari, D. P. Palomar, F. Facchinei, and P. Jong-Shi, "Convex Optimization, Game Theory, and Variational Inequality Theory," *IEEE Signal Process. Magazine*, vol. 27, pp. 35-49, 2010. [Article\(CrossRef Link\)](#)
- [24] V. Srivastava, J. Neel, A. B. MacKenzie, R. Menon, L. A. Dasilva, J. E. Hicks, *et al.*, "Using game theory to analyze wireless ad hoc networks," *IEEE Communications Surveys & Tutorials*, vol. 7, pp. 46-56, 2005. [Article\(CrossRef Link\)](#)
- [25] L. D. A.B.MacKenzie, and W. Tranter, "Game Theory for Wireless Engineers," *Morgan and Claypool Publishers*, 2006.
- [26] Beibei Wang, Yongle Wu, K.J. Ray Liu, "Game theory for cognitive radio networks: An overview," *Journal of Computer Networks*, 54, 2537-2561, 2010. [Article\(CrossRef Link\)](#)



- [27] Dimitris E. Charilas, Athanasios D. Panagopoulos, "A survey on game theory applications in wireless networks," *Journal of Computer Networks*, 2010. [Article\(CrossRef Link\)](#)
- [28] P. K. Dutta, "Strategise and Games Theory and Practice," 2001.
- [29] C. U. Saraydar, N. B. Mandayam, and D. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, pp. 291-303, 2002. [Article\(CrossRef Link\)](#)
- [30] G. Owen, "Game Theory," *New York, Academic*, 2001.
- [31] M. D. K. Frank H.P. Fitzek, "Cognitive Wireless Networks, Concepts, Methodologies and Vision, Inspiring the Age of Enlightenment of Wireless Communications," *Springer*, 2007.
- [32] Omar B. Abdulghafoor, Rosdiadee Norden, and Ayman Abd El-Saleh, "Resource Allocation in Spectrum Sharing ad-hoc Cognitive Radio Networks Based on Game Theory: An Overview," *KSII Transactions on Internet and Information Systems*, vol. 7, pp. 2957 - 2986, Dec. 2013. [Article\(CrossRef Link\)](#)
- [33] J. O. Neel, J. H. Reed, and R. P. Gilles, "Convergence of cognitive radio networks," in *Proc. of IEEE Conference on Wireless Commun. and Net., WCNC*, pp. 2250-2255, 2004. [Article\(CrossRef Link\)](#)
- [34] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Selected Areas in Commun.*, vol. 13, pp. 1341-1347, 1995. [Article\(CrossRef Link\)](#)
- [35] D. Niyato and E. Hossain, "Radio resource management games in wireless networks: an approach to bandwidth allocation and admission control for polling service in IEEE 802.16 [Radio Resource Management and Protocol Engineering for IEEE 802.16]," *Wireless Communications, IEEE*, vol. 14, pp. 27-35, 2007. [Article\(CrossRef Link\)](#)
- [36] D. N. Zhu Han, Walid Saad, Tamer Ba, sar, and Are Hjørungnes, "Game Theory in Wireless and Communication Networks: Theory, Models, and Applications," cambridge university press, 2012.
- [37] H. Zhu, Z. Ji, and K. J. R. Liu, "Fair Multiuser Channel Allocation for OFDMA Networks Using Nash Bargaining Solutions and Coalitions," *IEEE Trans. Commun.*, vol. 53, pp. 1366-1376, 2005. [Article\(CrossRef Link\)](#)
- [38] J. Nash, "The bargaining problem," *Econometrica*, vol. 18, pp. 155-162, Apr. 1950.
- [39] D. f. a. J.tirole, "Game Theory," *MIT Press*, 1991.
- [40] Y. Chun-gang, L. Jian-dong, and T. Zhi, "Optimal Power Control for Cognitive Radio Networks Under Coupled Interference Constraints: A Cooperative Game-Theoretic Perspective," *IEEE Trans. on Vehic. Tech.*, vol. 59, pp. 1696-1706, 2010. [Article\(CrossRef Link\)](#)
- [41] W. Saad, H. Zhu, M. Debbah, A. Hjørungnes, and T. Basar, "Coalitional game theory for communication networks," *IEEE Signal Process. Magazine*, vol. 26, pp. 77-97, 2009. [Article\(CrossRef Link\)](#)
- [42] Klemperer, "Auction theory: a guide to literature," *Journal of econometric suverys*, vol. 13, pp. 86-227, Mar. 1999. [Article\(CrossRef Link\)](#)
- [43] Z. Yang, L. Chonho, D. Niyato, and W. Ping, "Auction Approaches for Resource Allocation in Wireless Systems: A Survey," *IEEE Commun Surveys & Tut.*, vol. 15, pp. 1020-1041, 2013. [Article\(CrossRef Link\)](#)
- [44] Z. Yang, D. Niyato, P. Wang, and E. Hossain, "Auction-based resource allocation in cognitive radio systems," *IEEE Commun. Magazine*, vol. 50, pp. 108-120, 2012. [Article\(CrossRef Link\)](#)
- [45] Dusit Niyato, and Ekram Hossain, "Competitive Spectrum Sharing in Cognitive Radio Networks: A Dynamic Game Approach," *IEEE Trans. Wirel. Communi.*, Vol. 7, no. 7, July 2008. [Article\(CrossRef Link\)](#)
- [46] H. S. Mohammadian and B. Abolhassani, "Modeling spectrum sharing in cognitive radio networks by oligopolistic virtual market," in *Proc. of Conference on Electrical Engineering (ICEE)*, pp. 248-253, 2010. [Article\(CrossRef Link\)](#)

- [47] Ekram Hossain, Dusit Niyato and Zhu Han, "Dynamic Spectrum Access and Management in Cognitive Radio Networks," Cambridge University Press 2009
- [48] Ligia C. Cremene, D. Dumitrescu, and Réka Nagy, "Oligopoly Game Modeling for Cognitive Radio Environments," *Lecture Notes in Computer Science*, Volume 6294, pp 219-230, 2010.  
[Article\(CrossRef Link\)](#)
- [49] Dusit Niyato and Ekram Hossain, "A Game-Theoretic Approach to Competitive Spectrum Sharing in Cognitive Radio Networks," *Wireless Communications and Networking, WCNC*, Mar. 2007.  
[Article\(CrossRef Link\)](#)
- [50] Dusit Niyato and Ekram Hossain, "Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash Equilibrium, and Collusion," *IEEE Selected Areas Commun*, vol. 26, no. 1, Jan. 2008. [Article\(CrossRef Link\)](#)
- [51] Yuan Wu, Tengyi Zhang, and Danny H. K. Tsang, "Joint Pricing and Power Allocation for Dynamic Spectrum Access Networks with Stackelberg Game Model," *IEEE Trans. Wire. Communi.*, vol. 10, no. 1, Jan. 2011. [Article\(CrossRef Link\)](#)
- [52] Musbah Shaat and Faouzi Bader, "An Uplink Resource Allocation Algorithm for OFDM and FBMC Based Cognitive Radio Systems," in *Proc. of 5<sup>th</sup> International Conf. on CROWNCOM*, 2010
- [53] Hosseinali Jamal, Seyed Ali Ghorashi and Seyed Mohammad-Dajad Sadough, "A Fair Radio Resource Allocation Algorithm for Uplink of FBMC Based CR Systems," *KSII Trans. On Inter. Info. Sys.*, vol.6, no.6, June 2012. [Article\(CrossRef Link\)](#)
- [54] Dong Huang, Zhiqi Shen, Chunyan Miao and Cyril Leung, "Resource Allocatoin in MU-OFDM Cognitive Radio Systems with Partial Channel State Information," *EURASIP Journal on Wirel. Communi. Net.*, 2010. [Article\(CrossRef Link\)](#)
- [55] Tao Qin and Cyril Leung, "Fair Adaptive Resource Allocation for Multiuser OFDM Cognitive Radio Systems," in *Proc. of 2<sup>nd</sup> Inter. Conf. Communi. Net. CHINACOM*, China, 2007. [Article\(CrossRef Link\)](#)
- [56] Duy Trong Ngo, and Tho Le-Ngoc, "Distributed Resource Allocation for Cognitive Radio Networks With Spectrum-Sharing Constrians," *IEEE Transc. Veh. Tech.*, vol. 60, no. 7, Sept. 2011.  
[Article\(CrossRef Link\)](#)
- [57] Yonghong Zhang and Cyril Leung, "A Distributed Algorithm for Resource Allocation in OFDM Cognitive Radio Systems," *IEEE Trans. Veh. Tech.*, vo.60, no.2, Feb. 2011. [Article\(CrossRef Link\)](#)
- [58] Sanam Sadr, Alagan Anpalagan and Kaamaram Raahenmifar, "Radio Resource Allocation Algorithms for the Downlink of Multiuser OFDM Communications Systems," *IEEE Communi. Surv. Tut.*, Vol.11, no. 3, 2009. [Article\(CrossRef Link\)](#)
- [59] Guo-Peng Zhang, Ya-Li Zhong, En-Jie Ding, "Game Theoretic Subcarrier and Power Allocation for Wireless OFDMA Networks," *International Journal of Automation and Computing*, vol. 9, no. 4, Aug. 2012. [Article\(CrossRef Link\)](#)
- [60] Jae Cheol Park, Yun Hee Kim, Een Kee Hong and Lickho Song, "A Game Theoretic Power Control Algorithm With Sequential Subchannel Nulling for Wireless Networks", *IEICE, Trans. Commun.* Vol.E93-B, no.6. Jun.2010. [Article\(CrossRef Link\)](#)
- [61] Seyed Mahdi Mousavi Toroujeni, Seyed Mohammad-Sajad Sadough, and Seyed Ali Ghorashi, "Spectrum Leasing for OFDM-Based Cognitive Radio Networks," *IEEE Trans. Vehc. Tech.*, vol. 62, no. 5, Jun. 2013. [Article\(CrossRef Link\)](#)
- [62] Kai Yang, Narayan Prasad, and Xiaodong Wang, "An Auction Approach to Resource Allocation in Uplink OFDMA Systems," *IEEE Trans. Signal Proc.*, vol. 57, no. 11, Nov. 2009. [Article\(CrossRef Link\)](#)
- [63] Akram Baharlouei and Bijan Jabbari, "Efficient and Fair Power and Subchannel Allocation in Multiuser OFDM Networks," *IEEE Communi. Lett.*, Vol.17, no.10, Oct.2013. [Article\(CrossRef Link\)](#)

- [64] Haijian Zhang, Didier Le Ruyet, Daniel Roviras, and Hong Sun, "Noncooperative Multicell Resource Allocation of FBMC-Based Cognitive Radio Systems," *IEEE Trans Vehic. Tech.*, vol. 61, no. 2, Feb. 2012. [Article\(CrossRef Link\)](#)
- [65] D. Wu L. Zhou Y. Cai, "Energy-efficient resource allocation for uplink orthogonal frequency division multiple access systems using correlated equilibrium," *IET Communications*, 2012. [Article\(CrossRef Link\)](#)
- [66] Qiu Jing & Zhou Zheng, "Distributed Resource Allocation Based on Game Theory in Multi-cell OFDMA Systems," *Int J Wireless Inf Networks*, 2009. [Article\(CrossRef Link\)](#)
- [67] Quang Duy La, Yong Huat Chew and Boon Hee Soong, "An Interference-Minimization Potential Game for OFDMA-Based Distributed Spectrum Sharing Systems," *IEEE Trans. Vech. Tech.*, vol. 60, no. 7, Sept. 2011. [Article\(CrossRef Link\)](#)
- [68] Meixia Tao and Yuan Liu, "Spectrum Leasing and Cooperative Resource Allocation in Cognitive OFDMA Networks," *Journal Commun. Net.*, vol. 15, no. 1, Feb. 2013.
- [69] Aqeel Raza Syed and Kok-Lim Alvin Yau, "Spectrum Leasing in Cognitive Radio Networks: A Survey," *International Journal of Distributed Sensor Networks*, Feb. 2014. [Article\(CrossRef Link\)](#)
- [70] Dan Wu, YueMing Cai, YanMing Sheng, "Joint Subcarrier and Power Allocation in Uplink OFDMA Systems Based on Stochastic Game," *Science China Information Sciences*, vol.53, no.12, Dec.2010. [Article\(CrossRef Link\)](#)
- [71] Ioannis N. Stiakogiannakis · Dimitra I. Kaklamani, "A Radio Resource Management Framework for Multi-User Multi-Cell OFDMA Networks Based on Game Theory," *Wireless Pers Commun*, March 2013. [Article\(CrossRef Link\)](#)
- [72] Taskeen Nadkar, Vin Thumar, G.P.S. Tej, S.N Merchant, and U.B. Desai, "Distributed Power Allocation for Secondary Users in a Cognitive Radio Scenario," *IEEE Trans. Wire. Communi.*, vo.11 no.4, Apr.2012. [Article\(CrossRef Link\)](#)
- [73] C.K.Tan, T.C.Chuah, and S.W.Tan, "Fair subcarrier and power allocaiton for multiuser orthogonal frequency-division multiple access cognitive radio networks using a Colonel Blotto game," *IET Communi.*, 2011. [Article\(CrossRef Link\)](#)
- [74] Yiping Xing, R. Chandramouli and Carlos Cordeiro, "Price Dynamics in Competitive Agile Spectrum Access Markets," *IEEE Selec. Areas Commun.*, vol. 25, no. 3, Apr. 2007. [Article\(CrossRef Link\)](#)
- [75] Xiao Cheng wang, Xin Ping Guan, Qiao Ni Han, Zhi Xin Lin and Kai Ma, "A Stackelberg Game for Spectrum Leasing in Coopertive Cognitive Radio Networks," *Internat. J. Autumation & Computing*, Apr. 2013. [Article\(CrossRef Link\)](#)
- [76] Chao Xu, Min Sheng, Chungang Yang, Xijun Wang and Liang Wang, "Pricing Based Multi-Resource Allocation in OFDMA Cognitive Radio Networks: An Energy Efficiency Perspective," *IEEE Trans. Vich. Tech.*, Spet. 2013. [Article\(CrossRef Link\)](#)
- [77] Suchul Lee, Sangsoon Lim and Jun-Rake Lee, "An Oligopoly Spectrum Pricing with Behavior of Primary Users for Cognitive Radio Networks," *KSII Trasc. on Inter. And Info. Sys.*, Vol.8. no.4. Apr. 2014. [Article\(CrossRef Link\)](#)
- [78] Liang Liang, and Gang Feng, "A Game-Theoretic Framework for Interference Coordination in OFDMA Relay Networks," *IEEE TRANS. VEHIC. TECH.*, VOL. 61, NO. 1, JAN. 2012. [Article\(CrossRef Link\)](#)
- [79] Guftaar Ahmad Sardar Sidhu, Feifei Gao, Wei Wang, and Wen Chen, "Resource Allocation in Relay-Aided OFDM Cognitive Radio Networks," *IEEE TRANS. VEHIC. TECH.*, VOL. 62, NO. 8, OCT. 2013. [Article\(CrossRef Link\)](#)
- [80] Chiuian-Hsu Chen, Chin-Liang Wang, and Chih-Tsung Chen, "A Resource Allocation Scheme for Cooperative Multiuser OFDM-Based Cognitive Radio Systems," *IEEE TRANS, COMMUNI.*, VOL. 59, NO. 11, NOV. 2011. [Article\(CrossRef Link\)](#)

- [81] Musbah Shaat, and Faouzi Bader, "Asymptotically Optimal Resource Allocation in OFDM-Based Cognitive Networks with Multiple Relays," *IEEE TRANS. WIRELESS COMMUN.*, VOL. 11, NO. 3, MAR. 2012. [Article\(CrossRef Link\)](#)
- [82] Junling Mao, Gang Xie, Jinchun Gao, and Yuanan Liu, "Energy Efficiency Optimization for OFDM-Based Cognitive Radio Systems: A Water-Filling Factor Aided Search Method," *IEEE TRANS. WIRELESS COMMUN.*, VOL. 12, NO. 5, MAY 2013. [Article\(CrossRef Link\)](#)



**Omar B. Abdulghafoor** is a Ph.D. candidate in the Department of Electrical, Electronics and System Engineering, Universiti Kebangsaan Malaysia (UKM). He received his B.Sc. in Control and Computer Eng. from University of Technology, 2001 and the MEng. in Computer and Communication from Universiti Kebangsaan Malaysia in 2005. His research interests include cognitive radio, wireless communications, resource allocation and game theory applications.



**Mahamod Ismail** joined the Department of Electrical, Electronics and System Engineering, Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia (UKM) in 1985, and currently, he is a Professor in Communication Engineering. He is also appointed as the Deputy Director (Education and Research) Centre of Information Technology, UKM. He received the B.Sc. degree in Electrical and Electronics from University of Strathclyde, U.K. in 1985, the M.Sc. degree in Communication Engineering and Digital Electronics from University of Manchester Institute of Science and Technology (UMIST), Manchester U.K. in 1987, and the Ph.D. from University of Bradford, U.K. in 1996. He was with the first Malaysia Microsatellite TiungSat Team Engineers in Surrey Satellite Technology Ltd. U.K. for 9 months started in June 1997. His research interests include mobile and satellite communication as well as wireless networking, particularly, on the radio resource management for the next generation wireless communication. He also published more than 300 technical publications in local and international conferences and journals. He is an active member in professional bodies such as Institute of Electrical and Electronics Engineers (IEEE), USA and Malaysia Society of Engineers and Technologist (MSET) and currently the Vice Chair of IEEE Malaysia Section and executive committee member for Joint Communication and Vehicular Technology Society Chapter IEEE Malaysia.



**Rosdiadee Nordin** received his B.Eng. from Universiti Kebangsaan Malaysia in 2001 and Ph.D. from University of Bristol, United Kingdom in 2011. He is currently a senior lecturer in Department of Electrical, Electronics and System Engineering in Universiti Kebangsaan Malaysia. His research interests include resource allocation, green radio, Multiple-Input Multiple-Output (MIMO), Orthogonal Frequency-Division Multiple Access (OFDMA), carrier aggregation (CA), cognitive radio and indoor wireless localization.



**Musbah M Shaat** obtained his engineering degree major in communication and control from the Electrical and Computer Engineering Department at the Islamic University of Gaza (IUG), Palestine (1999-2004). He worked for one year (2004-2005) as full time teaching assistant at the electrical engineering department at the IUG. He awarded the DAAD "German Academic Exchange Service" master scholarship and got the master degree in communication and electronics engineering from Jordan University of Science and Technology (JUST), Jordan (2005-2007). He obtained his PhD degree from the Signal Theory and Communication department (TSC) at the Polytechnic University of Catalonia (UPC), Spain (2007-2012). He worked as a research associate in the Access Technologies Area in Centre Tecnologic de Telecomunicacions de Catalunya (CTTC)-Spain until he joined the electrical engineering department at the Islamic university of Gaza in September 2012.